Reading Questions 15

page 86: Definition 4.1

1. A group action is a set Ω.

2. A group action is a homomorphism σ.

3. Suppose G acts on the set Ω. Let a be the identity element in G and $\omega \in \Omega$. What is $a \cdot \omega$?

Section 4.1 Group Actions (Part 1)

Definition and Examples

P 1. Let $\Omega = \{\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\}\}$. Let $\sigma = (123)$. Then S_4 acts on Ω where $\sigma \cdot \{a,b\} = \{\sigma(a), \sigma(b)\}$. Compute $\sigma \cdot \{1,4\}$ and $\sigma \cdot \{2,3\}$.

P 2. Find a subgroup of S_4 which is isomorphic to Z_4. Hint Z_4 acts on $\{0,1,2,3\}$ where $g \cdot a = g + a$ mod 4.

P 3. Find a subgroup of S_4 which is isomorphic to D_6. Hint D_6 acts on $\{1,2,3\}$.

P 4. Let $G = GL(n, \mathbb{R})$ and let Ω be the set of all real $n \times n$ matrices. Let $A \in G$ and $B \in \Omega$. Define $A \cdot B = BAB^{-1}$. Show that G acts on Ω.

P 5. Let G be a group such that $H \leq G$. Prove or disprove: H acts on G where $h \cdot g = gh^{-1}$.