Reading Questions 9

page 55: Definition 2.51

page 56: Lemma 2.52 and its proof

1. A homomorphism is a map.
2. Let a and b be inverses of each other in some group. Then $ab = e$ by the cancellation property for groups.
3. Let $\phi : ({Z_5}^{\times}, \cdot) \to (Z_4, +)$ be a homomorphism. What is $\phi(1)$?

Section 2.4 Isomorphisms (Part 1)

Homomorphisms

P 1. Define a homomorphism from Z_4 to D_8.

P 2. Show that the map $\phi : Z_3 \to (Z_5)^{\times}$ such that $\phi(0) = 1, \phi(1) = 2$ and $\phi(2) = 4$ is not a homomorphism.

P 3. Give an example of an abelian group G, a non abelian group H, and a homomorphism $\phi : G \to H$.

Isomorphisms

P 4. Let $G = Z_4$ and $H = (Z_5)^{\times}$. Show that $G \cong H$.

> **Theorem**
>
> Let G and H be groups such that $\phi : G \to H$ is an isomorphism. Then G is abelian if and only if H is abelian. Moreover, $\forall x \in G, o(x) = m$ if and only if $o(\phi(x)) = m$ where m is a positive integer.

P 5. Use the previous theorem to show that D_8 is not isomorphic to Z_8.

P 6. Is Z_6 isomorhic to $(Z_7)^{\times}$?