

Reading Questions 7

page 45: Definition 2.27

page 45: Example 2.28

1. Let a be an element of a group. Then $a^{-2} = a^{-1}a^{-1}$.
2. Let a be an element of a group. Then $a^0 = 1$.
3. Let $a = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ be an element in $\text{GL}(2, \mathbb{Z}_3)$. Compute a^2 .

Section 2.3 Cyclic Groups and the Order of an Element (Part 1)

Cyclic Groups

- P 1.** Let $G = (\mathbb{Z}_5)^\times$ and $a = 2$. Compute a^3a^2 .
- P 2.** Let $G = \mathbb{Z}_5$ and $a = 2$. Compute a^3a^2 .
- P 3.** Let G be a group such that $a \in G$. Let $m \in \mathbb{Z}$ and $n = 0$. Prove $a^m a^n = a^{m+n}$.
- P 4.** Let G be a group such that $a \in G$. Let $m, n \in \mathbb{Z}$. Prove that $(a^n)^{-1} = a^{-n}$.
- P 5.** Show that $(\mathbb{Z}_7)^\times$ is cyclic by finding a generator for the group.
- P 6.** Determine if S_4 is cyclic.