Reading Questions 6

page 43: Definition 2.23

page 43: Theorem 2.24 and its proof

1. If a and b are elements of a semigroup and $ab = e$ where e is the identity element of the semigroup then a is a left inverse of b.
2. All semigroups are groups.
3. Do you have any concerns about the proof? Is the proof complete?

Section 2.2 Cancellation Properties (Part 1)

Properties of a Group

Definition

F a set S, a map $b : S \times S \to S$ is called a binary operation on S.

Definition

Assume that $\circ$ is an associative binary operation on a set G. Then $(G, \circ)$ is called a semigroup. In this case, we say G is a semigroup.

Theorem

Let G be an non-empty semigroup. Assume that G has a left identity and that every element of G has a left inverse. That is, there exists an element $e \in G$ such that, for every $a \in G$, $ea = a$, and, for every $a \in G$, there exists an element, denoted by a^{-1}, such that $a^{-1}a = e$. Then G is a group.

P 1. Let G be a group such that $a, b, c \in G$. Prove if $ba = ca$ then $b = c$.

P 2. Prove: Let G be a group. For all $a, b \in G$, $(ab)^{-1} = b^{-1}a^{-1}$.

More Properties of a Group

P 3. Show that $((Z_4)^{\times}, \cdot)$ is not a group.

P 4. Prove: The group G is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all $a, b \in G$.