Reading Questions 19

page 99: Example 4.40

1. The matrices A and B are similar if there exists a matrix P such that $AP = PB$. T
2. An elementary matrix is an invertible matrix. T
3. List the two actions given in the example for $P \cdot A$. PA, PAP^{-1}

Section 4.4 Orbits (Part 2)

More Examples

P 1. Show $\mathrm{GL}(n, \mathbb{R})$ acts on $\mathrm{M}_{n\times n}(\mathbb{R})$ where $P \cdot A = PA$ for $P \in \mathrm{GL}(n, \mathbb{R})$ and $A \in \mathrm{M}_{n\times n}(\mathbb{R})$.

P 2. Write $\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ as a product of elementary matrices.

P 3. What are the conjugacy classes of S_4?

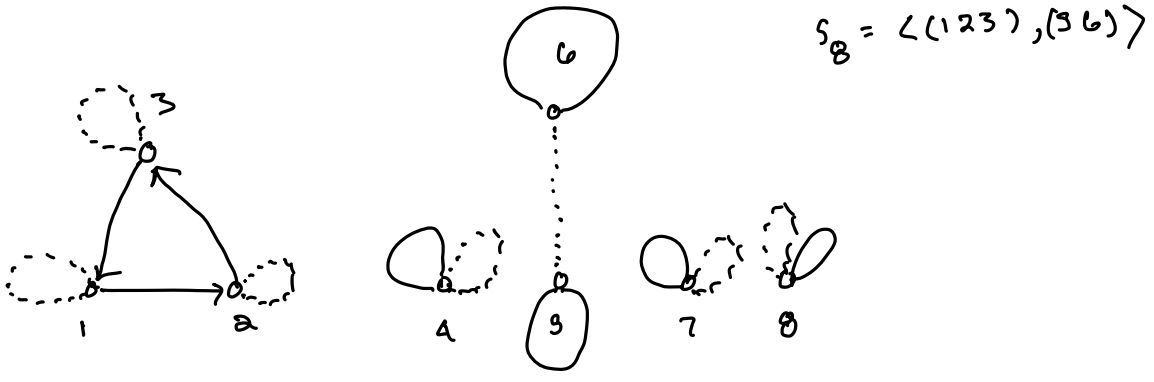
P 4. Let $(1432), (1324) \in S_4$. Find $\sigma \in S_4$ such that $(1432) = \sigma(1324)\sigma^{-1}$.

Ex: Let S_8 act on $\Omega = [8]$ in the natural way.

$g \cdot a = g(a)$

Then the cayley graph is given by (123) ——— , (56) - - - - -

$S_8 = \langle (123), (56) \rangle$



Hence the orbits are $O(1) = \{1,2,3\} = O(2) = O(3)$

$O(4) = \{4\}$ $O(5) = \{5,6\} = O(6)$

$O(7) = \{7\}$ $O(8) = \{8\}$

Def: Let G be a group such that $x \in G$. Then the conjugacy class of x in G is

$$cl_G(x) = \{ gxg^{-1} : g \in G \}$$

Ex: What are the conjugacy classes of S_3?

$$S_3 = \{ e, (12), (23), (13), (123), (132) \}$$

$cl_G(e) = \{e\}$ $\quad ge g^{-1} = gg^{-1} = e$

$cl_G((12)) = \{(12), (23), (13)\}$
$= cl_G(23) = cl_G(13)$

$cl_G((123)) = \{(123), (132)\}$
$= cl_G(132)$

if $gxg^{-1} = b$ then $O(b) = O(x)$ } by previous results

$(12) = g(23)g^{-1}$

$(12) \stackrel{?}{=} (13)(23)(13)$

$\stackrel{\checkmark}{=} (12)$