Reading Questions 16

page 88: Definition 4.9

page 88: Definition 4.10

1. A regular action is a group action. T
2. A translation action is a group action for a group G acting on itself.
3. Let G be a group and let H be a subgroup of G. Write the translation action.

Section 4.1 Group Actions (Part 2)

More Examples

P 1. Let $G = Z_4$. Use the regular action to compute $2 \cdot 3$.

P 2. Let $H \leq G$ such that $h \in H$ and $g \in G$. Does $h \cdot g = gh$ define a group action for H acting on G?

P 3. Does $g \cdot h = g^{-1}h$ define a group action for G on to G?

P 4. Let $G = Z_6$. Let $H =< 3 >$ and $g = 2$. Write the elements of gHg^{-1}.

$x \subseteq G$

$\langle x \rangle$

G acts on Ω

(0) $\forall g \in G \quad \forall \alpha \in \Omega \qquad g \cdot \alpha \in \Omega$

(1) $\forall \alpha \in \Omega \qquad e_G \cdot \alpha = \alpha$

(2) $\forall g, h \in G \qquad g \cdot (h \cdot \alpha) = gh \cdot \alpha$

Def. The group action of G on $\underline{G}$,
$g \cdot x = gx$ is called a regular action.

(0) $g \cdot x = gx \in G$

(1) $e_G \cdot x = e_G x = x$

(2) $h \cdot (g \cdot x) = h \cdot (gx) = hgx = hg \cdot x$

Def: The group action of $H \leq G$ on G, $h \cdot g = hg$ is called a translation action.

(0) $\forall h \in H \ \forall g \in G \quad h \cdot g = hg \in G$

(1) $e_H = e_G \Rightarrow e_H \cdot g = e_H g = e_G g = g$

(2) $\forall h_1, h_2 \in H \ \forall g \in G$

$$h_1 \cdot (h_2 \cdot g) = h_1 \cdot (\underset{\in G}{h_2 g}) = h_1 h_2 g$$

$$= h_1 h_2 \cdot g \qquad (H \leq G)$$

Def: The group action of G on G, $g \cdot h = ghg^{-1}$ is called a conjugate action.

(0) $g \cdot h = ghg^{-1} \in G$ (since G is a group)

(1) $e_G \cdot h = e_G h e_G^{-1} = e_G h e_G = e_G h = h$

(2) $g_1, g_2 \in G \quad h \in H$

$$g_1 \cdot (g_2 \cdot h) = g_1 \cdot g_2 h g_2^{-1}$$

$g_2 h g_2^{-1} \in G$ (G is a group)

$$= g_1 g_2 h g_2^{-1} g_1^{-1}$$

$g_2^{-1} g_1^{-1} = (g_1 g_2)^{-1}$

$$= g_1 g_2 h (g_1 g_2)^{-1}$$

$$= g_1 g_2 \cdot h$$

Def: Let G be a group such that $H \leq G$ and $g \in G$. Then $gHg^{-1} = \{ ghg^{-1} : h \in H \}$

Ex: Let $G = \mathbb{Z}_4$ and $H = \langle 2 \rangle$. Then

Let $1 = g$ $\quad g^{-1} = 3$ $\quad 1 + 3 = \overset{e_G}{\overset{\parallel}{0}} \mod 4$ $\quad H = \{2, 0\}$

$$1H3 = 1\{0,2\}3 = \{(1)(0)(3), (1)(2)(3)\}$$
$$= \{0, 2\}$$

Def: The group action of G on $K :=$ set of all subgroups of G

Then $g \cdot H = gHg^{-1}$ is called a conjugate action on the subgroups of G.

by HWP 2.6.27 ~~check~~

(0) $\forall g \cdot G$ $\quad H \in K$ $\quad g \cdot H = gHg^{-1} \in K$

(1) $e_G \cdot H = eHe^{-1} = eHe = H$ $\quad ehe = h$

(2) $g_1 \cdot g_2 \cdot H = g_1 \cdot (g_2 H g_2^{-1}) = g_1 g_2 H g_2^{-1} g_1^{-1}$

$$= g_1 g_2 H (g_1 g_2)^{-1}$$

$g_1 g_2 h g_2^{-1} g_1^{-1} = g_1 g_2 h (g_1 g_2)^{-1}$

$$= g_1 g_2 \cdot H$$