Reading Questions 13

page 75: Definition 3.1

- 1. A cycle in S_n of length m is a m- cycle. T
- 2. The cycles (123) and (231) are disjoint cycles. \mathbf{F}
- 3. What is the length of (123)(24). (124) -7 3

Section 3.1 Permutations, Cycles, and Transpositions (Part 1)

Cycles and Transpositions

P 1. List all 3 cycles in S_4 .

P 2. Give an example of 2 disjoint transpositions in S_6 .

P 3. If σ is a transposition, what is σ^{-1} ?

P 4. Prove the following statement. Let *n* be a positive integer. If σ and τ are disjoint cycles in S_n then $\sigma \tau = \tau \sigma$.

Properties

P 5. Write (123)(24)(321) as a product of disjoint cycles.

P 6. Write (1234)(231) as a product of transpositions.

P 7. A simple transposition is a transposition of the form (i i + 1). Write (1245) as a product of simple transpositions.

P 8. What is the order of (123)(25)(46) in S_7 ?

3.\

Recall
$$[n] := \{1, 2, ..., n\}$$

 $S_n = Perm([n]) = all bijections from [n] to [n]$

Def: Let
$$\sigma \in S_n$$
 such that $\sigma = (i_1, i_2, ..., i_m)$
where $i_1, i_2, ..., i_m$ are distinct. Then σ is an
m-cycle or a cycle of length *m*.

Ex: Let
$$\sigma = (123) \in S_{g}$$
. Then σ is a 3-cycle.

Des: The cycles
$$(i_1, \dots, i_m)$$
 and (j_1, \dots, j_e)
are disjoint if $\{i_1, \dots, i_m\} \cup \{j_1, \dots, j_e\} = \{\{j_1, \dots, j_e\}$.

Ex: The cycles (123) and (245) are not disjoint while (123) and (45) are disjoint cycles.

Def: A 2-cycle is called a transposition.

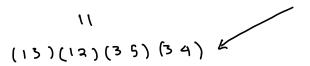
- (1) Every element in Sn can be written as a product of disjoint cycles.
 - pf: obvious since VoeSn ris a bijection.
 - (2) Every element in Sn can be written as a product of transpositions.

$$\mathcal{P}^{\mathcal{F}} = (i_{1}, i_{2}, \dots, i_{m}) \qquad i_{j} = i_{j+1} \quad j \leq m \quad i_{m} = i_{1}$$
Then
$$\mathcal{B} = (i_{1}, i_{m}) \quad (i_{1}, i_{m-1}) \quad \cdots \quad (i_{1}, i_{2})$$

$$\mathcal{B}(i_{1}) = i_{2} \qquad \mathcal{B}(i_{2}) = i_{3} \qquad \mathcal{B}(i_{j}) = i_{j+1} \quad j \leq m$$

$$\mathcal{B}(i_{m}) = i_{1}$$

E x<u>~</u>



What is

$$E_{X:}$$
 the order of $\sigma = (123)(25) = (1253)^2$. Hence
 $\sigma(\sigma) = 4$ by the previous thm.

Ex: What is the order of
$$\sigma = (123)(45)? \sigma$$
 is a cycle
decomposition so $\sigma(\sigma) = lon(3, 2) = 6$

$$(1253)^4 = e$$
 $(1253)^2 = (15)(23)$ 1 ciclength of σ
 $(1253)^3 = (1352)$ =? $\sigma(i) \neq i$

Hence if σ is a cycle then $o(\sigma) = \text{length of } \sigma$ if σ, τ are disjoint then $\sigma\tau = \tau\sigma$ $((123)(45))^6 = e$ $((123)(45))^2 = (123)(45)(123)(45)$ = (123)(123)(45)(45) $= (123)^2(45)^2$ $= (123)^2(45)^2$

$$((123)(45))^{3} = (123)^{3}(45)^{3}$$

= $(123)^{3}(45)^{2}(45)$ $((123)(45))^{4} = (123)^{4}(45)^{4}$
= $(123)^{2}(45)^{2} = (123)^{2}$

0 ((123)(45)) = 1cm (0(123), 0(45))