Reading Questions 2

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1. If f is a mapping from $\mathbb{Z}$ to $\mathbb{Z}$ and f maps all element in $\mathbb{Z}$ to 1 then f is an identity map.
2. Let f be a map. If f is $1-1$ and onto then f is a bijection.
3. If $f(3) = 4$ and $g(4) = 3$ then where does gf map 3?

Section 1.2 One to One and Onto Functions (Part 1)

Permutations

P 1. How many elements of $\mathrm{Perm}([1,2,3,4])$ map at least one element to itself?

P 2. Compute $|\mathrm{Perm}([1,2,3,\ldots,n])|$.

Cyclic Notation

P 3. Using cyclic notation write the elements of S_4.

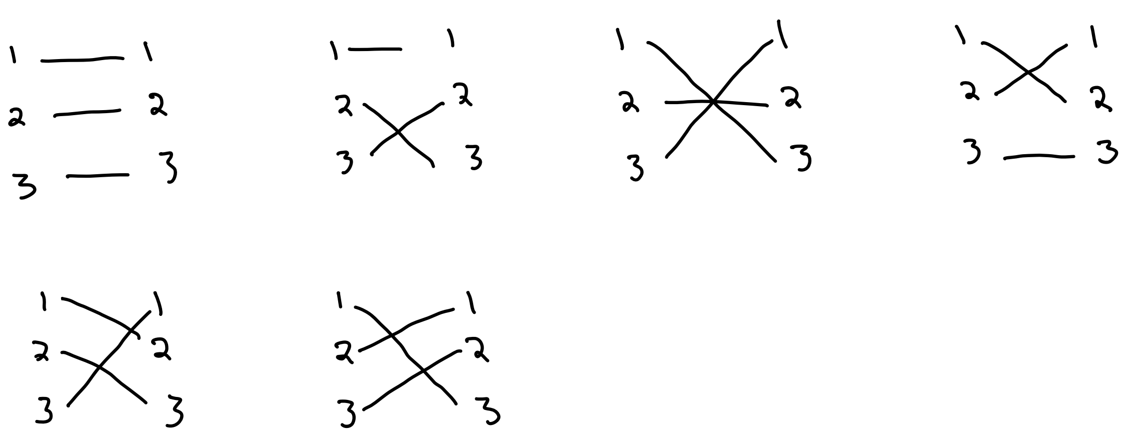
P 4. Write the elements of S_4 that contain a $2-$cycle.

1.2

Def: Let Ω be a set. Define

$$\mathrm{Perm}(\Omega) = \{ f : \Omega \to \Omega \text{ such that } f \text{ is 1-1 } \overset{\text{and}}{\&} \text{ onto} \}$$

Ex: $\mathrm{Perm}(\overset{\Omega}{\overset{\|}{[1,2,3]}})$ has 6 elements.



Def: A is finite

$|A| = \#$ of elements of A

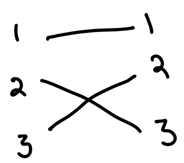
Ex: $\Omega = \{1, 2, 3\}$, $|\Omega| = 3$

$|Perm(\Omega)| = 6$

Def: Let n be a positive integer. Then

$$S_n = Perm([n]) \quad \text{where} \quad [n] = \{1, \dots, n\}.$$

Note: We saw $S_3 = Perm(\{1, 2, 3\})$.

$f:$  $:= \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$

$:= 132$ ← the last row of

$:= (1)(32)$ or (32) ← cyclic notation

Ex: Use cyclic notation to write $f = 4312$ and $g = 2431$ and $h = 2143$ in S_4.

$f = (3241) = (4132)$

$g = (3)(421) = (421) = (214)$

$h = (43)(21) = (12)(34) = (21)(34)$

Def: The cycle $(a_1, \dots, a_m)$ has length m and is an m-cycle. Here $a_1 \neq a_2 \neq \cdots \neq a_m$

Ex: The cycle $(2\,3)$ is a 2-cycle (transposition)

Def: If all the cycles of $f \in S_n$ are disjoint then the product is the cycle decomposition of f.

$$\overset{f_1}{(1\,2\,3)}\overset{f_2}{(4\,5)}\overset{f_3}{(4\,3\,2)} \qquad f = f_1 \circ f_2 \circ f_3 \overset{?}{=} f_2 \circ f_1 \circ f_3 \text{ check}$$

Ex: Let $f = (1\,2\,3)(4\,5)(4\,3\,2) \in S_5$

Then $(2)(4\,5\,3\,1) = (4\,5\,3\,1) = f = (1\,2\,3)(4\,5)(4\,3\,2)$.