TR - 2p

Reading Questions 1

page 4: the entire page

page 5: the entire page

1. The symmetries of a square are squares. F
2. Combining two symmetries of a square produces another symmetry. T
3. How many rotation symmetries are on the square? 4 $R_0, R_{90}, R_{180}, R_{270}$

Section 1.1 Symmetries of a Square (Part 1)

The table of D_8

P 1. Compute $R_{90}DVVHR_{180}$.

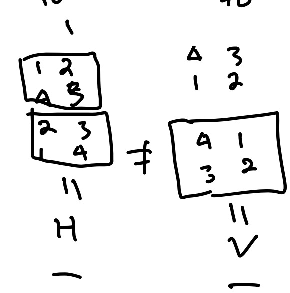
P 2. Compute the second row of the table for D_8. That is, compute $R_{180}X$ for all X in $\{R_0, R_{90}, R_{180}, R_{270}, H, V, D, D'\}$.

1 4
2 3

P 3. Find 2 elements x, y in D_8 such that $x \neq y$ and $xy \neq yx$. $R_{90}D \neq D R_{90}$

P 4. Find 2 elements x, y in D_8 such that $x = y$ and $xy = yx$.

$R_{90} R_{180}$



Alternative Notation

Definition: alternative definition of D_8

$$D_8 = < a, b \mid a^4 = b^2 = e, ba = a^3b >$$

P 5. Use the alternative definition of D_8 to compute ~~$R_{90}DVVHR_{180}$.~~ Write your final answer in terms of an element of D_8.

$R_{90} D V V H R_{90}$

Ex: The set of integers along with "+" form a group.

$3 \overset{\text{operation}}{+} 4 = 7$

elements

the object is 0

{ add 3 to 0 := 3
add 4 to 0 := 4

these are transformations

$$\begin{cases} \text{add } 3 \text{ to } 0 := 3 \\ \text{add } 4 \text{ to } 3 := 7 \end{cases}$$

this is composition of the transformations

$\{ \{ \ldots, -1, 0, 1, 2, \ldots \}, + \}$ is a group

D_8

$$R_0: \begin{matrix} 1 & 4 \\ 2 & 3 \end{matrix} \rightarrow \begin{matrix} 1 & 4 \\ 2 & 3 \end{matrix} \qquad R_{90}: \begin{matrix} 1 & 4 \\ 2 & 3 \end{matrix} \rightarrow \begin{matrix} 4 & 3 \\ 1 & 2 \end{matrix} \qquad R_{180}: \begin{matrix} 1 & 4 \\ 2 & 3 \end{matrix} \rightarrow \begin{matrix} 3 & 2 \\ 4 & 1 \end{matrix} \qquad R_{270}: \begin{matrix} 1 & 4 \\ 2 & 3 \end{matrix} \rightarrow \begin{matrix} 2 & 1 \\ 3 & 4 \end{matrix}$$

$$V: \begin{matrix} 1 & 4 \\ 2 & 3 \end{matrix} \rightarrow \begin{matrix} 4 & 1 \\ 3 & 2 \end{matrix} \qquad H: \begin{matrix} 1 & 4 \\ 2 & 3 \end{matrix} \rightarrow \begin{matrix} 2 & 3 \\ 1 & 4 \end{matrix} \qquad D: \begin{matrix} 1 & 4 \\ 2 & 3 \end{matrix} \rightarrow \begin{matrix} 1 & 2 \\ 4 & 3 \end{matrix} \qquad D': \begin{matrix} 1 & 4 \\ 2 & 3 \end{matrix} \rightarrow \begin{matrix} 3 & 4 \\ 2 & 1 \end{matrix}$$

$$D_8 = \{ \{ R_0, R_{90}, R_{180}, R_{270}, H, V, D, D' \}, \circ \}$$

<u>Def:</u> The operation of D_8, $\circ$, is defined by applying the symmetries (transformations) consecutively.

<u>Ex:</u> Compute $R_{90} V$.

$R_{90} \circ V$

$$\begin{matrix} 1 & 4 \\ 2 & 3 \end{matrix} \xrightarrow{V} \begin{matrix} 4 & 1 \\ 3 & 2 \end{matrix} \xrightarrow{R_{90}} \begin{matrix} 1 & 2 \\ 4 & 3 \end{matrix}$$

Therefore $R_{90} \circ V = D$

<u>Ex:</u> $f(x) = 2x + 1$

$g(x) = 3x + 2$

$f \circ g(3) = f(3(3) + 2)$

$= 2(3(3)+2) + 1$

Ex: Commpute $V R_{90} R_{90} H H D$.

Note:

$$HH = VV = DD = D'D' = R_0$$

Hence

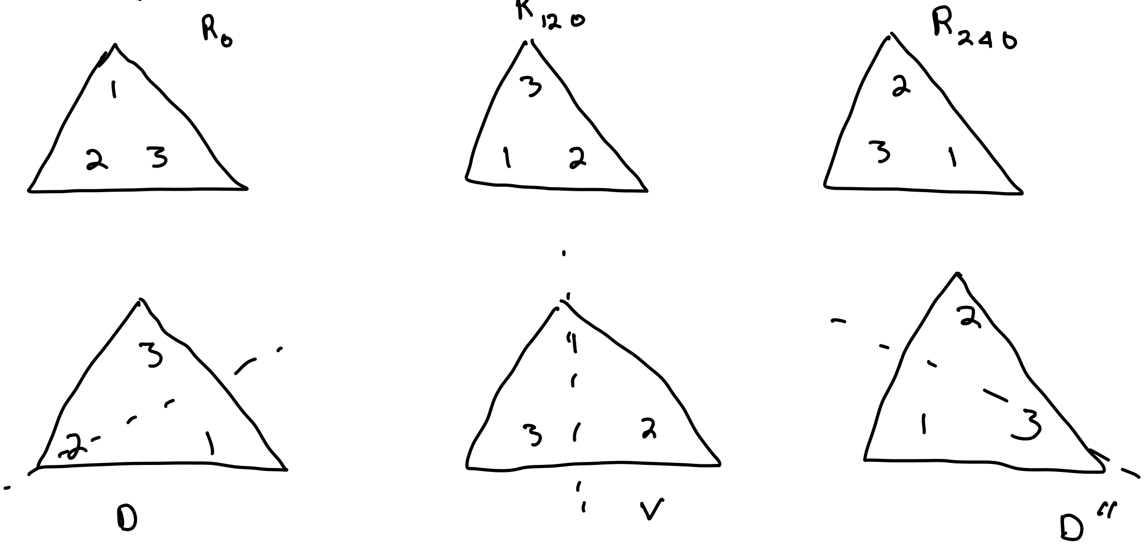
$$V R_{90} R_{90} HHD = V R_{90} R_{90} D \qquad R_{90} R_{90} = R_{180}$$

$$= V R_{180} D$$

$$\begin{matrix}1 & 4\\ 2 & 3\end{matrix} \xrightarrow{D} \begin{matrix}1 & 2\\ 4 & 3\end{matrix} \xrightarrow{R_{180}} \begin{matrix}3 & 4\\ 2 & 1\end{matrix} \xrightarrow{V} \begin{matrix}4 & 3\\ 1 & 2\end{matrix}$$

Therefore $V R_{90} R_{90} HHD = R_{90} \in D_8$

Ex: D_6



Def: D_{2n} = symmetries of a regular n-gon.

Notation: Consider D_8. Let $e = R_0$, $a = R_{90}$ and $b = H$.

Next time we will see that

$$D_8 = \{\overset{R_0}{e}, \overset{R_{90}}{a}, \overset{R_{180}}{a^2}, \overset{R_{270}}{a^3}, \overset{H}{b}, \overset{V}{a^2 b}, \overset{D'}{ab}, \overset{D}{a^3 b}\}$$

$$= \langle a, b \mid a^4 = b^2 = e,\ \underline{ba = a^3 b} \rangle$$

$R_{90} D = a$

$R_{90} = a$

$R_{90} R_{90} = R_{180} = a\,a = a^2$

$R_{270} = a^3$

$R_0 = a^4 = e$

$R_{90} D = a\, a^3 b$

$= a^4 b$

$= eb = b = H$

$\begin{array}{cc} 1 & 4 \\ 2 & 3 \end{array}$

$ab = D'$

$R_{90} H = D'$

$\begin{array}{cc} 2 & 3 \\ 1 & 4 \end{array}$

$\begin{array}{cc} 3 & 4 \\ 2 & 1 \end{array}$

$R_{90} D' = a\,a\,b$

$= a^2 b = V$

$\begin{array}{cc} 3 & 4 \\ 2 & 1 \end{array}$

$\begin{array}{cc} 4 & 1 \\ 3 & 2 \end{array} = V$

$D R_{90} = a^3 b a$

$= a^3 a^3 b$

$= a^4 a^3 b = e a^2 b$

$= a^2 b$

$= V$