Reading Questions 11

page 136: example 1

1. If $\vec{v}, \vec{w} \in \mathbb{R}^n$ and the first entry of $\vec{v}$ is nonzero and the first entry of $\vec{w}$ is zero then $\vec{v}$ and $\vec{w}$ are linearly independent.
2. If $\text{span}(\vec{v}_2, \vec{v}_2, \vec{v}_3) = \ker(A)$ then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a basis for $\ker(A)$.
3. If $\text{span}(\vec{v}_2, \vec{v}_2) = \ker(A)$ and $\vec{v}_1$ and $\vec{v}_2$ are linearly independent then the dimension of $\ker(A)$ is 2.
4. Suppose $\text{span}(\begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}) = V$. What is the dimension of V?

Section 3.3 The Dimension of a Subspaces in R^n (Part 1)

Dimension

P 1. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 3 & 6 \end{bmatrix}$.

1. Find a basis for the kernel.
2. Find a basis for the image.
3. Determine the dimensions for each of the previously found subspaces.
4. Use the dimension of the image of A to determine the number of free variables for the system $A\vec{x} = \vec{0}$.
5. Use the dimension of the kernel of A to determine the rank of A.

P 2. For which values of the constant k do the following vectors form a basis for $\mathbb{R}^3$?

$$\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ k \\ k^2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

P 3. Consider the plane $2x_1 + 3x_2 + x_3 = 0$ which is a subspace of $\mathbb{R}^3$.

1. Find a matrix whose kernel is the same as the plane.
2. Find a basis for the plane.
3. Find the dimension of the plane.