Reading Questions 10

page 124: example 4

1. The image of a matrix is the set of columns of the matrix.
2. If $\vec{v} = \vec{u} + 3\vec{w}$ then $\text{span}(\vec{v}, \vec{u}, \vec{w}) = \text{span}(\vec{v}, \vec{w})$.
3. What is the image of I_4?

Section 3.2 Subspaces of R^n (Part 1)

The Kernel and the Image

P 1. Show that the line $x_1 + x_2 = 0$ in $\mathbb{R}^2$ is a subspace of $\mathbb{R}^2$.

P 2. Determine if the following set is a subspace of $\mathbb{R}^3$. Be sure to justify your answer.

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x + y + z = 1 \right\}$$

P 3. Let T be a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$. Show that the kernel of T is a subspace of $\mathbb{R}^n$.

Bases and Linear Independence

P 4. Write down one way of determining if a set of vectors are linearly independent.

P 5. Determine if the vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$ are linearly independent.

P 6. Find a basis for the image of the matrix

$$A = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -5 & 4 \\ 3 & -2 & 1 & -2 \end{bmatrix}.$$

Write the redundant vectors as a linear combination of the basis vectors.

P 7. Find a basis for the kernel of the matrix

$$A = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -5 & 4 \\ 3 & -2 & 1 & -2 \end{bmatrix}.$$