Reading Questions 9

page 117: example 11

1. If T is a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ then the kernel of T is the set of vectors in $\mathbb{R}^n$ which get mapped to the zero vector in $\mathbb{R}^m$.
2. The kernel of a linear transformation can be found by solving the linear system $A\vec{x} = \vec{1}$.
3. What is the kernel of I_3?

Section 3.1 The image and kernel of a linear transformation (Part 1)

The Image

P 1. Fill in the blank.

1. The _____ of a function $f : X \to Y$ is the set of values the function takes in its target space.
2. If $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m$ are in $\mathbb{R}^n$. Then the set

$$\{c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_m\vec{v}_m : c_1, \ldots, c_m \in \mathbb{R}\}$$

is called _____.

P 2. Write the image of the linear transformation $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$.

The Kernel

P 3. Find the vectors that span the image of $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$.

P 4. Write down the definition of the kernel of a linear transformation.

P 5. Find a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ whose image is the line spanned by the vector $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$.

P 6. Find the vectors that span the kernel of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

P 7. Assume A is a $n \times m$ matrix. If $\text{rank}(A) = m$ what is the kernel of A?

P 8. Find a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ whose kernel is the line spanned by the vector $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$.