Reading Questions 8

page 94: example 3

page 95: theorem 2.4.9

1. If $A = \begin{bmatrix} a & 1 \\ 0 & d \end{bmatrix}$ is invertible then a and d are both not 0.
2. The determinant of $A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ is 5.
3. Write down one way to determine if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible.

Section 2.4 The Inverse of a Linear Transformation (Part 2)

Inverse of linear transformation proofs

P 1. Write down the inverse of the product of two invertible matrices A and B.

P 2. Show that the inverse of a matrix A is unique.

P 3. Find all matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $ad - bc = 1$ and $A^{-1} = A$. Here $ad - bc$ is called the determinant of A.

P 4. Write down the determinant of the following matrices. Explain how they are related.

$$\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}, \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}, \begin{bmatrix} a & 0 \\ c & d \end{bmatrix}$$

These matrices are upper triangular, diagonal, and lower triangular matrices.

P 5. Without using row operations, find the inverse of $\begin{bmatrix} 1 & 4 & 4 \\ 0 & 2 & 1 \\ 0 & 2 & 4 \end{bmatrix}$ given that $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 2/3 & -4/3 \\ 0 & 2/3 & -1/3 \\ 0 & -1/3 & 2/3 \end{bmatrix}$.