Reading Questions 6

page 79: theorem 2.3.4

page 79: example 1

1. If A is an $n \times p$ matrix and B is an $p \times m$ matrix then BA is an $p \times p$ matrix.
2. The ith entry of BA is a dot product.
3. Compute $\begin{bmatrix} 2 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$. You may write your matrix using the notation

   [1 3]
   [2 4]

   to represent the matrix $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$.

Section 2.3 Matrix Product (Part 1)

Multiplying Matrices

P 1. Let A and B be the $n \times m$ and $m \times p$ matrices respectively. What is the size of AB?

P 2. Compute $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. In general, does $AB = BA$?

P 3. Let the matrix representation of the linear transformations T and S be

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 2 & 2 & 1 \\ 3 & 2 & 1 & 1 \end{bmatrix}$$

respectively. Find the matrix representation of $T \circ S$.

Multiplying Matrices

P 4. Compute the following product of matrices $\begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$.

P 5. Find a 3×3 matrix A which is not I_3 or $-I_3$ such that $AA = I_3$.

P 6. Suppose $T(\vec{x})$ rotates $\vec{x}$ counterclockwise by θ, and $S(\vec{x})$ rotates $\vec{x}$ counterclockwise by $-\theta$.

1. Find the matrix A of T and the matrix B of S.
2. Compute AB.
3. Interpret $(T \circ S)(\vec{x})$ geometrically.

P 7. Let A, B, and C be $n \times n$ matrices. Show that $A(B + C) = AB + AC$.

P 8. Find a 2×2 matrix A such that $A^2 \neq I$ and $A^4 = I$.

P 9. Let P be the matrix projection onto $\vec{u} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$. Is there a matrix Q such that $QP = I$?