Reading Questions 3

page 26: example 2

page 28: definition 1.3.5

1. The notation rank(A) represents the number of nonzero entries in the rref(A).
2. The sum of an $n \times n$ matrix A and $n \times n$ matrix B is an $n \times n$ matrix.
3. List the entries of b if $b = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

Section 1.3 On the Solutions of Linear Systems (Part 1)

The rank of a matrix

P 1. For each of the following augmented matrices write its solutions and state the number of solution it has.

$$A = \begin{bmatrix} 1 & 0 & | & 5 \\ 0 & 1 & | & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 2 & 1 & | & 0 \\ 0 & 1 & 3 & 1 & | & 0 \\ 0 & 1 & 3 & 1 & | & 0 \\ 0 & 1 & 3 & 1 & | & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 3 \end{bmatrix}$$

P 2. For each of the following matrices write the rref and determine its rank.

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$$

P 3. Suppose that A is an $n \times n$ coefficient matrix and the rank of A is n. How many solutions does the system of equations have? Justify your answer.

Matrix Algebra

P 4. Compute

$$\begin{bmatrix} 1 & 2 & 2 \\ 3 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 2 \\ 3 & 2 & 3 \\ 2 & 3 & 2 \end{bmatrix}, \quad 4\begin{bmatrix} 1 & 2 & 2 \\ 3 & 2 & 3 \\ 2 & 3 & 2 \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 & 2 \\ 3 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}.$$

P 5. Compute the product $A\vec{x}$ by using the rows of A.

$$A = \begin{bmatrix} 4 & 4 & 2 \\ 5 & 5 & 1 \\ 3 & 3 & 1 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

P 6. Let $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 3 & 2 \\ 1 & 3 & 1 \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$. Write $\begin{bmatrix} 13 \\ 15 \\ 12 \end{bmatrix}$ as a linear combination of the columns of A.