Reading Questions 18

Example 7.2.1

- 1. The characteristic equation of a matrix A can be used to determine the eigenvalues of the matrix A.
- 2. To find the eigenvalues of a 3×2 matrix we need to solve a quadratic equation.
- 3. The algebraic multiplicity of an eigenvalue is the number of times it appears in the list of all eigenvalues. T WSelf
- 4. Give your reasoning for your answer of the previous problem.

Section 7.2 Finding the eigenvalues of a matrix (Part 1)

Characteristic Equation

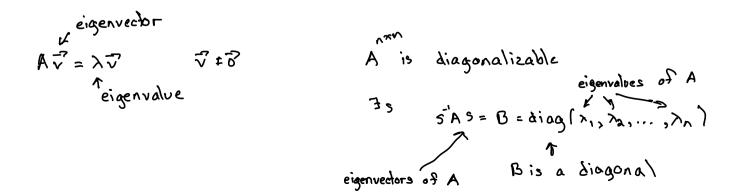
P 1. Let A be an $n \times n$ matrix.

- 1. Write down the characteristic equation of A.
- 2. Write down the characteristic polynomial of A where $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigenvalues of A.
- **P** 4. What does it mean for a matrix A to have an eigenvalue of 2 with algebraic multiplicity 3.
- **P** 5. Find the eigenvalues for the matrix

-3	0	0	0	0	0
0	2	0	0	0	$\begin{bmatrix} 0\\0 \end{bmatrix}$
0	0	1	$ \begin{array}{c} 0 \\ 0 \\ 0 \\ 3 \\ 0 \end{array} $	0	0
0	0	0	3	0	0
0	0	0		1	$\begin{array}{c} 0 \\ 1 \\ 2 \end{array}$
0	0	0	0	1	2
-					_

and their algebraic multiplicities.

P 6. Let $A = \begin{bmatrix} 1 & k \\ 1 & 1 \end{bmatrix}$ where k is some real number. For which values of k does A have two distinct real eigenvalues?



Thm: Let A be nxn. Then
$$\lambda$$
 is an eigenvalue of
A IF and only if $\det(A - \lambda I) = 0$.
P
Suppose $A\vec{x} = \lambda \vec{x}$ where $\vec{x} \pm \vec{0}$.
Then $A\vec{x} - \lambda \vec{x} = \vec{0}$
 $F = 7 (A - \lambda I)\vec{x} = \vec{0}$
 $\vec{x} \in \ker A - \lambda I$
 $\vec{x} \in \ker A - \lambda I$
 $\vec{x} \in \ker A - \lambda I$
 $\vec{x} \in \ker A - \lambda I = 0$
 $4et (A - \lambda I) = 0$
Def: $\det(A - \lambda I) = 0$ is the characteristic equation
for A

Ex: Find the eigenvalues for $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.

det (A - XI) = O

$$A - \lambda I = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1 - \lambda & 2 \\ 4 & 3 - \lambda \end{bmatrix}$$

 $def \quad A - \lambda I = (1 - \lambda)(3 - \lambda) - (2)(4)$ $= \lambda^{2} - 4\lambda - 5$ $def \quad A - \lambda I = 0$ $= 7 \qquad \lambda^{2} - 4\lambda - 5 = 0$ $(\lambda - 5)(\lambda + 1) = 0 = 7 \qquad \lambda = -1, 5$

Thm: If A is upper triangular then the diagonal entries are the eigenvalues of A.

Def: Let A be nxn. Then
$$\lambda_0$$
 has algebraic
multiplicity is and only if
 $f_A(\lambda) = (\lambda_0 - \lambda)^K g(\kappa)$ where $g(\lambda_0) \neq 0$.

$$B = \operatorname{diag}(4, 3, 1, 2) = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

if $A = SBS^{-1}$

eigenvalues of A: A, 3, 1, 2 eigenvectors of A: columns of S rank of A: A : :