Reading Questions 18

Example 7.2.1

1. The characteristic equation of a matrix A can be used to determine the eigenvalues of the matrix A. T
2. To find the eigenvalues of a $\underline{3} \times \underline{2}$ matrix we need to solve a <u>quadratic equation.</u> F
3. The algebraic multiplicity of an eigenvalue is the number of times it appears in the list of all eigenvalues. T by def
4. Give your reasoning for your answer of the previous problem.

Section 7.2 Finding the eigenvalues of a matrix (Part 1)

Characteristic Equation

P 1. Let A be an $n \times n$ matrix.

1. Write down the characteristic equation of A.
2. Write down the characteristic polynomial of A where $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigenvalues of A.

P 2. Find the eigenvalues for the matrix $\begin{bmatrix} -3 & 0 & 4 \\ 0 & -1 & 0 \\ -2 & 7 & 3 \end{bmatrix}$.

P 3. Find the eigenvalues for the matrix $\begin{bmatrix} 3 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 7 & 3 \end{bmatrix}$.

P 4. What does it mean for a matrix A to have an eigenvalue of 2 with algebraic multiplicity 3.

P 5. Find the eigenvalues for the matrix

$$\begin{bmatrix} -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

and their algebraic multiplicities.

P 6. Let $A = \begin{bmatrix} 1 & k \\ 1 & 1 \end{bmatrix}$ where k is some real number. For which values of k does A have two distinct real eigenvalues?

eigenvector

$A\vec{v} = \lambda \vec{v}$ $\quad \vec{v} \neq \vec{0}$

eigenvalue

$A^{n \times n}$ is diagonalizable

eigenvalues of A

$\exists S$ $\quad S^{-1}AS = B = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$

eigenvectors of A $\qquad$ B is a diagonal

7.2

Thm: Let A be $n \times n$. Then λ is an eigenvalue of A if and only if $\det(A - \lambda I) = 0$.

pf: Suppose $A\vec{x} = \lambda \vec{x}$ where $\vec{x} \neq \vec{0}$.

Then $A\vec{x} - \lambda\vec{x} = \vec{0}$

$\iff (A - \lambda I)\vec{x} = \vec{0}$

$\iff \vec{x} \in \ker A - \lambda I$

$\iff \ker A - \lambda I \neq \{\vec{0}\}$

$\iff A - \lambda I$ is not invertible

$\iff \det(A - \lambda I) = 0$

Def: $\overbrace{\det \; A - \lambda I}^{\text{polyn}} = 0$ is the characteristic equation for A

Ex: Find the eigenvalues for $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.

$\det(A - \lambda I) = 0$

$$A - \lambda I = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{bmatrix}$$

$$\det A - \lambda I = (1-\lambda)(3-\lambda) - (2)(4)$$
$$= \lambda^2 - 4\lambda - 5$$

$$\det A - \lambda I = 0$$

$$\Rightarrow \quad \lambda^2 - 4\lambda - 5 = 0$$

$$(\lambda - 5)(\lambda + 1) = 0 \quad \Rightarrow \quad \lambda = -1, 5$$

Thm: If A is upper triangular then the diagonal entries are the eigenvalues of A.

Def: Let A be $n \times n$. Then λ_0 has algebraic multiplicity k if and only if

$$f_A(\lambda) = (\lambda_0 - \lambda)^k g(k) \quad \text{where} \quad g(\lambda_0) \neq 0.$$

Ex:

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$2, 0$ are eigen val

2 has alg mult 2

0 has alg mult 3

$$B = \text{diag}(4, 3, 1, 2) = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

if $A = SBS^{-1}$

eigenvalues of A : 4, 3, 1, 2

eigenvectors of A : columns of S

rank of A : 4

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