Reading Questions 16

page 283: example 5

1. If A and B are similar matrices and $\det(A^6) = 5$ then $\det(B^5) = \frac{1}{5}$.
2. For any square matrices A and B it is the case that $\det(AB) = \det(A)\det(B)$.
3. Suppose $\det(A) = 6$ and S is an invertible matrix. What is $\det(S^{-1}AS)$?

Section 6.2 Properties to determinants (Part 1)

The Transpose

P 1. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix}$. Compute $\det(A^T)$.

P 2. Let $A = \begin{bmatrix} 1 & 4 & 6 & 8 \\ 1 & 2 & 3 & 8 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 6 & 8 \end{bmatrix}$. Compute the determinant of A using elementary row operations and determine if A is invertible.

P 3. Given some numbers a, b, c, d, e and f such that

$$\det \begin{bmatrix} a & 1 & d \\ b & 1 & e \\ c & 1 & f \end{bmatrix} = 7,$$

find

$$\det \begin{bmatrix} a & 3 & d \\ b & 3 & e \\ c & 3 & f \end{bmatrix}.$$

Similar Matrices

P 4. Let $A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix}$. Suppose A and B are similar matrices. Find $\det(B)$ and $\det(A^{-1})$.

P 5. If A can be reduced to I by applying exactly 8 row additions what is the determinant of A.

P 6. Compute the determinant of A^{-1}, A^4 and $(A^2)^T$ from problem 2.

6.2

Def: The transpose of an $n \times m$ matrix A, denoted by A^T, is an $m \times n$ matrix whose rows are the columns of A.

Ex: Let $A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 4 \end{bmatrix}$. Then $A^T = \begin{bmatrix} 3 & 1 \\ 2 & 2 \\ 1 & 4 \end{bmatrix}$

Ex: Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

— = inverted

--- = not inverted

$A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

$P_A = P_{A^T} \Rightarrow \text{prod } P_A = \text{prod } P_{A^T}$

# of inversions for P_A and P_{A^T} is the same

since x, y is inverted in P_A

if and only if y, x is inverted in P_{A^T}

Thm: Let A be $n \times n$. Then $\det A = \det A^T$

Thm: suppose B is obtained from A by

1. dividing a row of A by k then $\det B = \frac{1}{k} \det A$
2. swapping two rows of A then $\det B = -\det A$
3. adding a multiple of a row of A to another row of A then $\det A = \det B$

Ex: Find $\det A$ where $A = \begin{bmatrix} 0 & 7 & 5 & 3 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 2 & -1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$

$$\det A = -\det \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 7 & 5 & 3 \\ 1 & 1 & 2 & -1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \overset{\text{by (2)}}{=} -\det \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 7 & 5 & 3 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & -1 & 1 \end{bmatrix} \overset{\text{by (3)}}{}$$

$$\overset{\text{by (2)}}{=} (-1)(-1)\det \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 7 & 5 & 3 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$$= (-1)^2 (1)(7)(-1)(-2) = 14$$

Thm: Let A be a invertible $n \times n$. Then $\det A = (-1)^s k_1 \cdot k_2 \cdot \cdots \cdot k_r \det(\text{rref } A)$

$$= (-1)^s k_1 \cdot k_2 \cdot \cdots \cdot k_r$$

where s is the number of row swaps and

k_i are the scalar divisions.

Thm: Let A and B be $n \times n$ matrices.

Then $\det AB = \det A \det B$

Thm: If A and B are similar then $\det A = \det B$

Thm: If A is invertible then $\det A^{-1} = \dfrac{1}{\det A}$.

pf. :

$$\det A A^{-1} = \det I$$

$$\det A \det A^{-1} = 1$$

$$\det A^{-1} = \frac{1}{\det A}$$