Reading Questions 15

page 269-270

1. A pattern is a matrix. ~~T~~ F
2. The diagonal entries of a square matrix is a pattern. T
3. Compute 5!. $120 = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

Section 6.1 Introduction to determinants (Part 2)

Determinants and Patterns

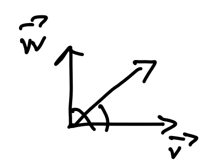
P 1. Compute the determinant for the follow matrices by using their patterns.

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 2 & 2 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 1 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

P 2. Compute the determinant for the matrix M.

$$M = \begin{bmatrix} 0 & 0 & 1 & 0 & 2 \\ 5 & 4 & 3 & 2 & 1 \\ 1 & 3 & 5 & 0 & 7 \\ 2 & 0 & 4 & 0 & 6 \\ 0 & 0 & 3 & 0 & 4 \end{bmatrix}$$

P 3. Find $\det(A^2)$ for $A = \begin{bmatrix} 1 & 81 & 80 & 88 \\ 0 & 2 & 86 & 84 \\ 0 & 0 & 3 & 87 \\ 0 & 0 & 0 & 4 \end{bmatrix}$



6.1

<u>Def:</u> A pattern, denoted by P, of an $n \times n$ matrix A is a list of n entries of A such that there is exactly one entry from every row and column.

<u>Ex:</u>

Let $A = \begin{bmatrix} 1 & 19 \\ 3 & 4 \end{bmatrix}$. The list $P_1 = [1, 4] = [a_{11}, a_{22}]$

$P_2 = [\,3, 19\,]$.

<u>Def:</u> Two entries in a pattern are inverted if one of them is located to the right and above the other in the matrix.

<u>Ex:</u> Let $A = \begin{bmatrix} 1 & 19 \\ 3 & 4 \end{bmatrix}$. $P_1 = [1, 4]$ ← no inverted entries

$P_2 = [3, 19]$ has one inversion

3 & 19 are inverted

$$A = \begin{bmatrix} 1 & \underline{2} & 3 \\ \underline{4} & 5 & 6 \\ 2 & 1 & \underline{3} \end{bmatrix} \qquad P = [4, 2, 3]$$

4 & 2 are inverted

<u>Def:</u> The product of a pattern P, denoted by prod(P), is the product of the entries of the pattern P.

<u>Ex:</u> $P = [4, 2, 3]$ $\quad \text{prod}(P) = 4 \cdot 2 \cdot 3$

<u>Def:</u> Let A be $n \times n$. Then

$$\det A = \sum_{P} (-1)^{(\#\text{ of inversion of } P_i)} \text{prod}(P_i)$$

Ex:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

P_1 $\qquad$ P_2 $\qquad$ P_3

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

P_4 $\qquad$ P_5 $\qquad$ P_6

\# of inversions

P_1 – 0

P_2 – 1

P_3 – 1

P_4 – 2 $\quad a_{21}$ & a_{13} , a_{32} & a_{13}

P_5 – 2

P_6 – 3

$$\det A = (-1)^0 a_{11}a_{22}a_{33} + (-1)^1 a_{11}a_{23}a_{32} + (-1)^1 a_{21}a_{12}a_{33} + (-1)^2 a_{21}a_{32}a_{13} + (-1)^2 a_{31}a_{12}a_{23} + (-1)^3 a_{31}a_{22}a_{13}$$

$$\det A = (a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23}) - (a_{11}a_{23}a_{32} + a_{21}a_{12}a_{33} + a_{31}a_{22}a_{13}) \checkmark$$

check

Ex: Find det(A) for $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ $\qquad$ 5.

$P_1 = [2, 4]$ - 0 $\qquad$ $P_2 = [1, 3]$ - 1

$$\det A = (-1)^0 \cdot 2 \cdot 4 + (-1)^1 \cdot 1 \cdot 3 = 8 - 3 = 5 \quad \checkmark$$

Ex: Find det A where $A = \begin{bmatrix} \underline{5} & 0 & 1 & 0 & 0 \\ 9 & \underline{3} & 2 & 3 & 7 \\ 8 & 0 & 3 & \underline{2} & 9 \\ 0 & 0 & \underline{4} & 0 & 0 \\ 5 & 0 & 5 & 0 & \underline{1} \end{bmatrix}$

There is only one pattern that doesn't contain a zero.

$P_1 = [5, 3, 4, 2, 1]$ - 1 inversion

$\det A = (-1)^1 \cdot 5! = -120$ $\qquad A^{-1}$ exists

Ex: Find det A where $A = \begin{bmatrix} \underline{1} & 0 & 0 & 8 \\ 0 & \underline{2} & 6 & 4 \\ 0 & 0 & \underline{3} & 7 \\ 0 & 0 & 0 & \underline{9} \end{bmatrix}$

$\det A = (-1)^0\, 4!$ $\qquad$ no inversion