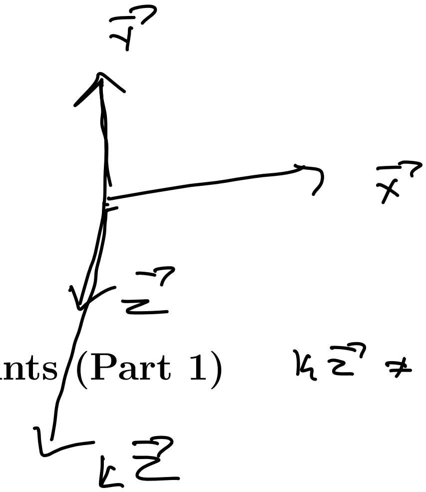
Reading Questions 14

page 463: definition a.9

1. The cross product $\vec{v} \times \vec{w}$ is a unit vector. F
2. If $\vec{u}$ is orthogonal to $\vec{v}$ and $\vec{w}$ then $\vec{u} = \vec{v} \times \vec{w}$. F
3. Give a vector in $\mathbb{R}^4$ which is orthogonal to both $\vec{e}_1$ and $\vec{e}_2$.

$\vec{e}_1 \times \vec{e}_2$



Section 6.1 Introduction to determinants (Part 1)

$k\vec{z} \neq \vec{z}$

Cross Product

P 1. Write down the formula for the determinant of a 3×3 matrix.

P 2. If the 3×3 matrix A is not invertible then the determinant of A is ______.

P 3. If the 3×3 matrix A is not invertible then the dimension of the image of A is ______.

P 4. Compute the determinant for the follow matrices.

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 2 & 2 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 1 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

P 5. Compare $\det(A), \det(B), \det(C)$ and $\det(D)$.

Recall: $A^{n\times n}$

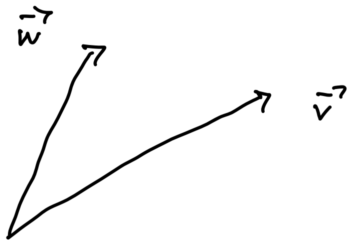
A^{-1} exists $\Rightarrow$
- $\det A = 0$ $(A^{2\times 2})$
- $A\vec{x} = \vec{b}$ has a unique solution
- $\operatorname{rank} A = n$
- $\ker A = \{\vec{0}\}$
- $\operatorname{rref} A = I_n$
- $\operatorname{im} A = \mathbb{R}^n$
- the columns of A are linearly independent

$A\vec{x} = \vec{b}$ what is $\vec{x}$?

$\vec{x} = A^{-1}\vec{b}$

$$A = \begin{bmatrix} | & | & | \\ \vec{u} & \vec{v} & \vec{w} \\ | & | & | \end{bmatrix}$$

Assume $\vec{v}$ & $\vec{w}$ are L.I.



If A is not invertible then $\vec{u} \cdot (\vec{v} \times \vec{w}) = 0$.

<u>Thm:</u> Let $A = \begin{bmatrix} | & | & | \\ \vec{u} & \vec{v} & \vec{w} \\ | & | & | \end{bmatrix}$. Then A is invertible if and only if $\vec{u} \cdot (\vec{v} \times \vec{w}) \neq 0$

Here $\vec{u} \cdot (\vec{v} \times \vec{w}) = \det A :=$ determinant of $A^{3\times 3}$

<u>Sarrus Rules</u>

$$\begin{array}{ccccc} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{array}$$

$\diagup$ = $-$

$\diagdown$ = $+$

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = -\left(a_{13}a_{22}a_{31} + a_{11}a_{23}a_{32} + a_{12}a_{21}a_{33}\right)$$
$$+\left(a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}\right)$$

$$\vec{u} = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix} \qquad \vec{w} = \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix}$$

$$\vec{v} \times \vec{w} = \begin{bmatrix} a_{22}a_{33} - a_{23}a_{32} \\ a_{32}a_{13} - a_{12}a_{33} \\ a_{12}a_{23} - a_{22}a_{13} \end{bmatrix}$$

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} \cdot \begin{bmatrix} a_{22}a_{33} - a_{23}a_{32} \\ a_{32}a_{13} - a_{12}a_{33} \\ a_{12}a_{23} - a_{22}a_{13} \end{bmatrix}$$

$$= (a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32}) + (a_{21}a_{32}a_{13} - a_{21}a_{12}a_{33}) + (a_{31}a_{12}a_{23} - a_{31}a_{22}a_{13})$$

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = -(a_{13}a_{22}a_{31} + a_{11}a_{23}a_{32} + a_{12}a_{21}a_{33}) + (a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32})$$

Ex: Find det A where $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix} \begin{matrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{matrix}$

$$\det A = 1\cdot 5\cdot 10 + 2\cdot 6\cdot 7 + 3\cdot 4\cdot 8 - (7\cdot 5\cdot 3 + 8\cdot 6\cdot 1 + 10\cdot 4\cdot 2)$$

$\neq 0 \Rightarrow A$ is invertible.