Reading Questions 13

page 156: example 7

1. The matrix $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ is similar to the matrix $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$. S is not invertible , F
2. If $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ is similar $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ then $\begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$ is similar to $\begin{bmatrix} 4 & 2 \\ 0 & 1 \end{bmatrix}$.
3. Suppose $A = SBS^{-1}$ where $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$. What is B? $\underline{S^{-1}AS = B} \Rightarrow S^{-1}$

Section 3.4 Coordinates (Part 2)

$A = B$ see board

The coordinate vector

P 1. Write down what it means for two matrices to be similar.

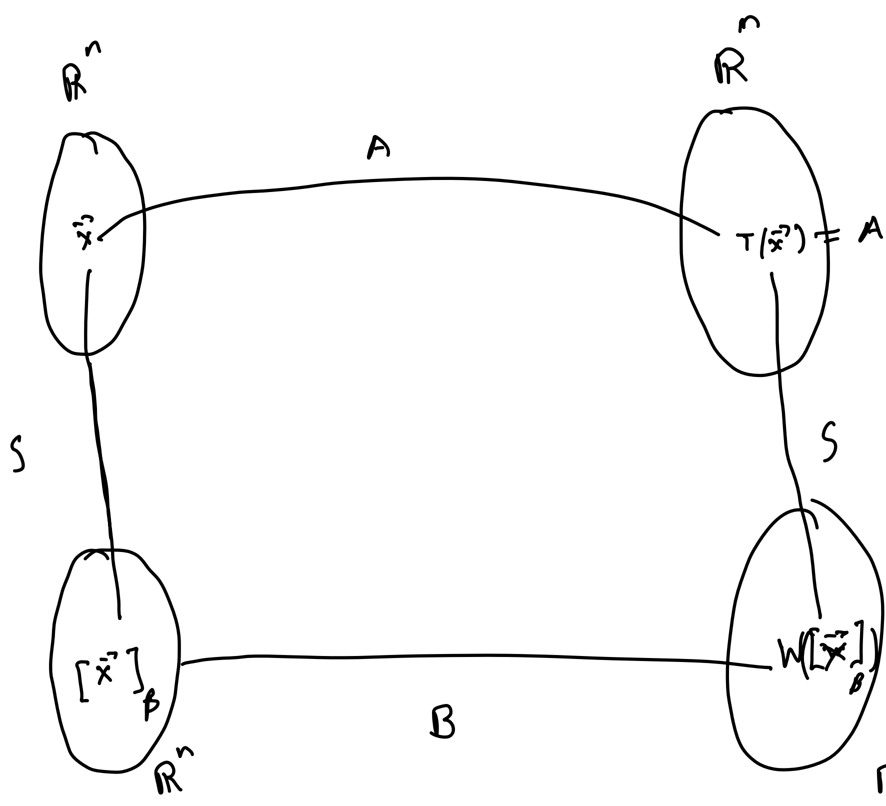
P 2. If A is similar to the identity matrix what can you say about A?

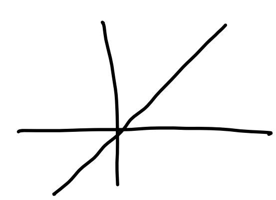
P 3. Let

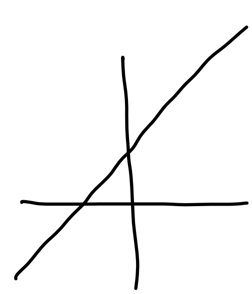
$$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}.$$

Find an invertible matrix S such that $AS = SB$.

P 4. Suppose $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} = S \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix} S^{-1}$. Find a matrix C which is similar to $\begin{bmatrix} 9 & 8 \\ 16 & 17 \end{bmatrix}$.







Thm: Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a LT such that $T(\vec{x}) = A\vec{x}$.

Let B be a $\mathcal{B}$-matrix of T. Then

$$AS = SB \quad \text{where} \quad S = \begin{bmatrix} | & & | \\ \vec{v}_1 & \cdots & \vec{v}_n \\ | & & | \end{bmatrix}$$

$\mathcal{B} = \{\vec{v}_1, \ldots, \vec{v}_n\}$. In this case we say A is similar to B.

Ex: Are the matrices $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}$ similar?

$$\underbrace{\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{S} = \underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{S} \underbrace{\begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}}_{B}$$

$$\begin{bmatrix} a+2c & b+2d \\ 4a+3c & 4b+3d \end{bmatrix} = \begin{bmatrix} 5a & -b \\ 5c & -d \end{bmatrix}$$

$\Rightarrow$ $a + 2c = 5a \qquad 2c = 4a \qquad c = 2a$

$4a + 3c = 5c$

$4a + 3 \cdot 2a = 5 \cdot 2a$

$4a + 6a = 10a$

$10a = 10a$

$0 = 0 \qquad$ let $a = 1$ then $c = 2$

$b + 2d = -b \Rightarrow 2d = -2b \Rightarrow d = -b$

$4b + 3d = -d \Rightarrow 4b = -4d \Rightarrow d = -b$

let $b = 1$ then $d = -1$

$$S = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \qquad ad - bc \neq 0 \Rightarrow S^{-1} \text{ exists!}$$

$\therefore$ A and B are similar.

Ex: Show that if A is similar to B then A^t is similar to B^t for all positive integer.

Let t be a positive integer. Since A is similar to B there exists an invertible S suchthat $AS = SB$. Hence

$$A \underbrace{S S^{-1}}_{I} = S B S^{-1}$$

$$A = S B S^{-1}$$

$$A^t = (S B S^{-1})^t = \underbrace{S B \overbrace{S^{-1} S}^{I} B \overbrace{S^{-1} \cdots}^{I} \overbrace{\cdots S}^{I} B S^{-1}}_{t \text{ times}}$$

$$= S \underbrace{B \cdot B \cdots B}_{t\text{-times}} S^{-1}$$

$$= S B^t S^{-1}$$

$\Rightarrow A^t S = S B^t$. $\therefore$ A^t is similar to B^t.