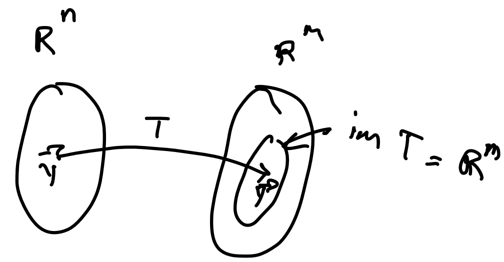


Reading Questions 10

page 124: example 4

$\vec{v} = -\vec{v} + 3\vec{w}$

1. The image of a matrix is the set of columns of the matrix. F
2. If $\vec{v} = \vec{u} + 3\vec{w}$ then $\text{span}(\vec{v}, \vec{u}, \vec{w}) = \text{span}(\vec{v}, \vec{w})$. T
3. What is the image of I_4?

$= \text{span}(\vec{v}, \vec{w})$

$I_4 \vec{v} = \vec{y}$

$\text{span}(\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4) = R^4$

Section 3.2 Subspaces of R^n (Part 1)

The Kernel and the Image

P 1. Show that the line $x_1 + x_2 = 0$ in $\mathbb{R}^2$ is a subspace of $\mathbb{R}^2$.

P 2. Determine if the following set is a subspace of $\mathbb{R}^3$. Be sure to justify your answer.

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x + y + z = 1 \right\}$$

P 3. Let T be a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$. Show that the kernel of T is a subspace of $\mathbb{R}^n$.

Bases and Linear Independence

P 4. Write down one way of determining if a set of vectors are linearly independent.

P 5. Determine if the vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$ are linearly independent.

P 6. Find a basis for the image of the matrix

$$A = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -5 & 4 \\ 3 & -2 & 1 & -2 \end{bmatrix}.$$

Write the redundant vectors as a linear combination of the basis vectors.

P 7. Find a basis for the kernel of the matrix

$$A = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -5 & 4 \\ 3 & -2 & 1 & -2 \end{bmatrix}.$$

$\text{rref}[A : \vec{0}]$

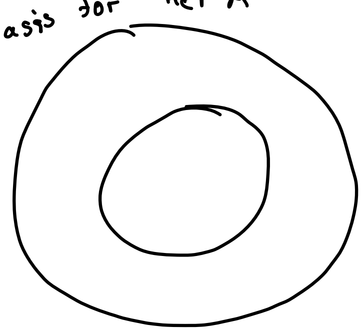
$$\begin{matrix} 1 & 0 & -3 & 2 & 0 \\ 6 & 1 & -5 & 4 & 0 \\ 0 & -2 & 10 & -8 & 0 \end{matrix} \Rightarrow \begin{matrix} 1 & 0 & -3 & 2 & 0 \\ 0 & 1 & -5 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$\text{span}\left(\begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \ker(A)$

$\left\{\begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}\right\}$ is a basis for $\ker A$

$x_1 = 3x_3 - 2x_4$

$x_2 = 5x_3 - 4x_4 \Rightarrow \vec{x} = x_3 \begin{bmatrix} 3 \\ 5 \end{bmatrix} + x_4 \begin{bmatrix} -2 \\ -4 \end{bmatrix}$



$A = \{1, 2, 3\}$ - a set

$B = \{1, 2\}$ - B is a subset of A

3.2

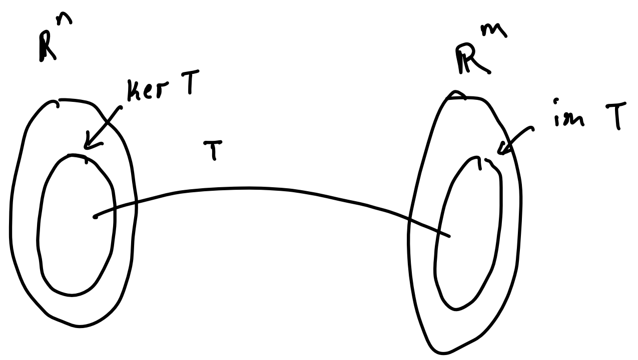
vector space

Def: A subset W of $\mathbb{R}^n$ is a subspace of $\mathbb{R}^n$ if

1. $\vec{0} \in W$

2. If $\vec{w}_1, \vec{w}_2 \in W$ then $\vec{w}_1 + \vec{w}_2 \in W$

3. If $\vec{w}_1 \in W$ and $k \in \mathbb{R}$ then $k\vec{w}_1 \in W$.



Thm: If T is a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$

then

1) $\text{im}(T)$ is a subspace of $\mathbb{R}^m$

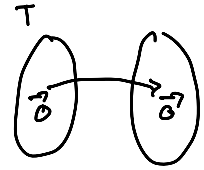
2) $\ker(T)$ is a subspace of $\mathbb{R}^n$.

pf: (1) WTS $\vec{0} \in \text{im}(T)$

if $\vec{v}, \vec{u} \in \text{im}(T)$ and $k \in \mathbb{R}$

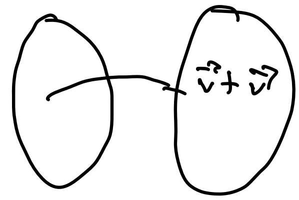
then $\vec{v} + \vec{u} \in \text{im } T$ and $k\vec{v} \in \text{im } T$

$\vec{0} \in \text{im}(T)$ $T(\vec{0}) = \vec{0}$ which shows that $\vec{0} \in \text{im } T$



$\vec{v}+\vec{u} \in \operatorname{im} T$

$T = A \qquad A^{-1}$ ← does it exist?



$\vec{v}, \vec{u} \in \operatorname{im} T \Rightarrow$ there exists $\vec{a}, \vec{b} \in \mathbb{R}^n$ such that

$T(\vec{a}) = \vec{v}$ and $T(\vec{b}) = \vec{u}$

$\vec{v} + \vec{u} = T(\vec{a}) + T(\vec{b})$. Since T is a LT

$T(\vec{a}) + T(\vec{b}) = T(\vec{a}+\vec{b})$. So

$\vec{v} + \vec{u} = T(\vec{a}+\vec{b})$ which shows that $\vec{v}+\vec{u} \in \operatorname{im} T$.

$k\vec{v} \in \operatorname{im} T$

Since $\vec{v} \in \operatorname{im} T$ there exists $\vec{a} \in \mathbb{R}^n$ such that

$T(\vec{a}) = \vec{v}$. So $k\vec{v} = kT(\vec{a})$. Since T is a LT

$kT(\vec{a}) = T(k\vec{a})$. Hence $k\vec{v} = T(k\vec{a})$.

This shows that $k\vec{v} \in \operatorname{im} T$.

$\square$

Ex: Let $W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x, y \geq 0 \right\}$. Then $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \in W$

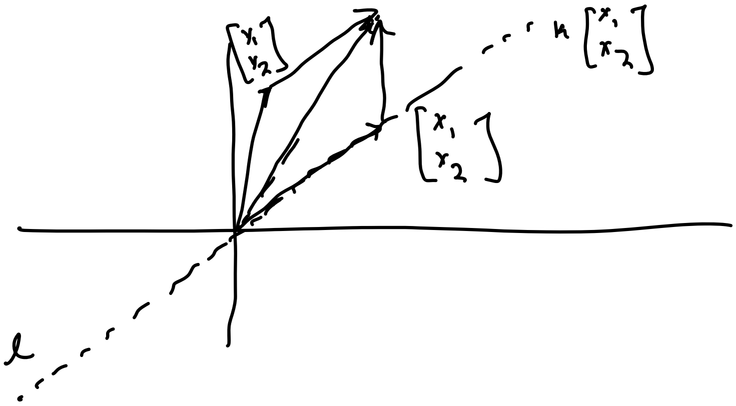
but $-1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \notin W$. By (3) W is not a subspace of $\mathbb{R}^2$.

Ex: Let W be a subspace of $\mathbb{R}^2$.

Then W can be $\{\vec{0}\}$.

Suppose W is not $\{\vec{0}\}$ and $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in W$.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$



3) $k\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in W$

W is l or W is R^2

$k_1\vec{x} + k_2\vec{x} = (k_1+k_2)\vec{x}$

$\{\vec{0}\} = \text{span}(\vec{0})$

$l = \text{span}(\vec{x}_1)$

$R^2 = \text{span}(\vec{x}_1, \vec{x}_2)$