If A is invertible $A\vec{x}=\vec{b}$ has a unique solution.

$\vec{x} = A^{-1}\vec{b}$

Reading Questions 7

$[A \;\vdots\;]$

page 89: theorem 2.4.4

page 90: example 1

1. If $\overset{A}{\cancel{B}}$ is an invertible matrix then the linear system $A\vec{x} = \vec{1}$ has infinitely many solutions. F by 2.4.4
2. If A is not an invertible matrix then the linear system $A\vec{x} = \vec{b}$ has no solution. F
3. If the rref$(\begin{bmatrix} A & \mid & I_4 \end{bmatrix}) = \begin{bmatrix} 1 & 0 & 0 & \mid & 2 & 3 & 0 \\ 0 & 1 & 0 & \mid & 3 & 0 & 1 \\ 0 & 0 & 1 & \mid & 0 & 3 & 2 \end{bmatrix}$ what is the inverse of A?

Section 2.4 The Inverse of a Linear Transformation (Part 1)

Inverse of linear transformations

P 1. Let $A = \begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$. Suppose $\vec{b} \in \mathbb{R}^2$. How many solutions does the linear equation $A\vec{x} = \vec{b}$ have?

P 2. Let A be an $n \times n$ matrix.

1. Write down one way of determining if the inverse of a matrix A exists.
2. Explain why $\text{rank}(A) = n$ if A is invertible.
3. If A is invertible what is $\text{rank}(A^{-1})$?

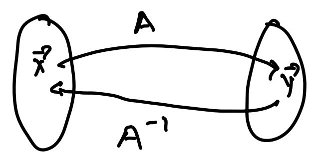
P 3. Find B^{-1} where $B = \begin{bmatrix} 4 & 10 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

P 4. Verify that $BB^{-1} = B^{-1}B = I_3$.

P 5. Solve the following linear system using B^{-1}.

$$\left| \begin{array}{rcr} 4x_1 + 10x_2 & = & 7 \\ x_1 + 3x_2 & = & -5 \\ x_3 & = & 4 \end{array} \right|$$

2.4 Inverses for square matrices



Def:

A LT $T(\vec{x}) = A\vec{x} = \vec{y}$ is invertible if the linear system has a unique solution for all $\vec{y}$ in $\mathbb{R}^n$.

Ex: Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. ~~What is the inverse of A?~~

Let $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$. Then

$$\text{rref}\left[\, A \mid \vec{b} \,\right] = \left[\begin{array}{cc|c} 1 & 0 & b_1 - b_2 \\ 0 & 1 & b_2 \end{array}\right]$$

Since $b_1 - b_2$ and b_2 are real numbers

$A\vec{x} = \vec{b}$ has a unique solution.

Hence A is invertible.

Def: A^{-1} is the inverse of A

Facts: $A^{n \times n}$

(1) if A^{-1} exists then $\text{rref}(A) = I_n$

(2) if A^{-1} exists then $\text{rank}(A) = n$

Thm: Let A be $n \times n$. If $\text{rref}\,[A : I_n] = [I_n : B]$ then $A^{-1} = B$. Otherwise A is not invertible.

Ex: Let $A = \begin{bmatrix} 1 & -3 & 4 \\ 2 & -5 & 7 \\ 0 & -1 & 2 \end{bmatrix}$.

1) Find A^{-1}.

$$\text{rref}\left[\begin{array}{ccc|ccc} 1 & -3 & 4 & 1 & 0 & 0 \\ 2 & -5 & 7 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array}\right] = \text{rref}\begin{bmatrix} 1 & -3 & 4 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

$$= \text{rref}\begin{bmatrix} 1 & 0 & 1 & -5 & 3 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{bmatrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & -1 \\ 0 & 1 & 0 & -4 & 2 & 1 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -3 & 2 & -1 \\ -4 & 2 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

(2) check $A^{-1}A = AA^{-1} = I$

$$\begin{bmatrix} 1 & -3 & 4 \\ 2 & -5 & 7 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} -3 & 2 & -1 \\ -4 & 2 & 1 \\ -2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

try $A^{-1}A$

(3) solve $A\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

If $A\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

then $A^{-1}A\vec{x} = A^{-1}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow I_n\vec{x} = A^{-1}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \vec{x} = \begin{bmatrix} -3 & 2 & -1 \\ -4 & 2 & 1 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} -3 \\ -4 \\ -2 \end{bmatrix}$