Reading Questions 6

page 79: theorem 2.3.4

page 79: example 1

1. If A is an $n \times p$ matrix and B is an $p \times m$ matrix then BA is an $p \times p$ matrix. F

2. The ith entry of BA is a dot product. T

3. Compute $\begin{bmatrix} 2 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$. You may write your matrix using the notation

   [1 3]
[2 4]

   $\begin{bmatrix} 2 & 8 \\ 2 & 4 \end{bmatrix}$

   to represent the matrix $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$.

Section 2.3 Matrix Product (Part 1)

Multiplying Matrices

P 1. Let A and B be the $n \times m$ and $m \times p$ matrices respectively. What is the size of AB?

P 2. Compute $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. In general, does $AB = BA$?

P 3. Let the matrix representation of the linear transformations T and S be

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 2 & 2 & 1 \\ 3 & 2 & 1 & 1 \end{bmatrix}$$

respectively. Find the matrix representation of $T \circ S$.

$f \circ g(x) = f(g(x))$

Multiplying Matrices

P 4. Compute the following product of matrices $\begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$.

P 5. Find a 3×3 matrix A which is not I_3 or $-I_3$ such that $AA = I_3$.

P 6. Suppose $T(\vec{x})$ rotates $\vec{x}$ counterclockwise by θ, and $S(\vec{x})$ rotates $\vec{x}$ counterclockwise by $-\theta$.

1. Find the matrix A of T and the matrix B of S.
2. Compute AB.
3. Interpret $(T \circ S)(\vec{x})$ geometrically.

P 7. Let A, B, and C be $n \times n$ matrices. Show that $A(B + C) = AB + AC$.

P 8. Find a 2×2 matrix A such that $A^2 \neq I$ and $A^4 = I$.

P 9. Let P be the matrix projection onto $\vec{u} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$. Is there a matrix Q such that $QP = I$?

2.4

Thm: Let T and S be LT such that $T : \mathbb{R}^p \to \mathbb{R}^n$ and $S : \mathbb{R}^m \to \mathbb{R}^p$. Then $T \circ S : \mathbb{R}^m \to \mathbb{R}^n$ is a LT.

To show T is a LT

1) $T(\vec{x} + \vec{y}) = T(\vec{x}) + T\vec{y}$

2) $k\,T(\vec{x}) = T(k\vec{x})$

pf:

1) Let $\vec{x}, \vec{y} \in \mathbb{R}^m$ and $k \in \mathbb{R}$. Then

$$T \circ S(\vec{x} + \vec{y}) \overset{\text{by def}}{=} T(S(\vec{x} + \vec{y}))$$
$$\overset{S-LT}{=} T(S(\vec{x}) + S(\vec{y}))$$
$$\overset{T-LT}{=} T(S(\vec{x})) + T(S(\vec{y}))$$
$$= T \circ S(\vec{x}) + T \circ S(\vec{y}) \quad \checkmark$$

$$k\, T \circ S(\vec{x}) \overset{\text{by def}}{=} k\, T(S(\vec{x}))$$
$$\overset{T-LT}{=} T(k\, S(\vec{x})) \overset{S-LT}{=} T(S(k\vec{x}))$$
$$= T \circ S(k\vec{x})$$

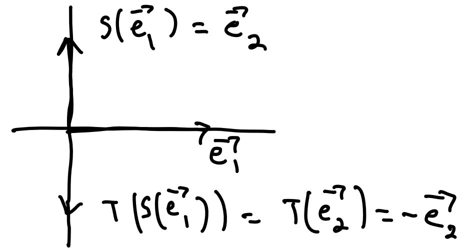
Ex: Let $T(\vec{x}) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\vec{x}$ & $S(\vec{x}) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\vec{x}$

cos 90   - sin 90

sin 90   cos 90

$$T \circ S\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = T\left(S\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$
$$= \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

gives the first column of the $T \circ S$ matrix



$$T \circ S\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = T\left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

the second column of the $T \circ S$ matrix

$$T \circ S(\vec{x}) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \vec{x}$$

$f : X \to Y$

$g : X \to Y$

$f = g$ if $\forall_{x \in X}$

$f(x) = g(x)$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{x} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \vec{x}$$

$$\Rightarrow \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Thm:

$$A = \begin{bmatrix} | & & | \\ \vec{v}_1 & \cdots & \vec{v}_m \\ | & & | \end{bmatrix}$$

$$BA = \begin{bmatrix} | & & | \\ B\vec{v}_1 & \cdots & B\vec{v}_m \\ | & & | \end{bmatrix}$$

Ex: Let $T(\vec{x}) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \vec{x}$ and $S(\vec{x}) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{x}$.

Then the matrix for $(T \circ S)(\vec{x})$ is

$$\left[\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix} \right]$$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Rules for matrix product

- $A^{n \times p}$ $B^{q \times m}$ then $(AB)^{n \times m}$ if and only if $p = q$

- associative $(AB)C = A(BC)$

- $AB \neq BA$ always

- distributive $(A+B)C = AC + BC$

- $A^k = \underbrace{A \cdot A \cdot A \cdot \dots \cdot A}_{k\text{-times}}$