Reading Questions 4

page 33: theorem 1.3.10

Page 42-44 excluding Example 1,2

1. If A is an $n \times m$ matrix, $\vec{x}, \vec{y} \in \mathbb{R}^m$, and k is a real number then $A(k\vec{x} + \vec{y}) = kA\vec{x} + A\vec{y}$. T
2. All coding transformations don't have an inverse. F
3. Suppose the position of my boat is 6° Eastern latitude and 10° Northern latitude. Use the following code to determine my encoded position.

$$\left| \begin{array}{rcl} x_1 + 2x_2 & = & y_1 \\ 2x_1 + x_2 & = & y_2 \end{array} \right|$$

$$\left.\begin{array}{l} (6) + 2(10) = y_1 \\ 2(6) + (10) = y_2 \end{array}\right\} \Rightarrow \begin{array}{l} y_1 = 26 \\ y_2 = 22 \end{array}$$

Section 2.1 Linear Transformations and Their Inverse (Part 1)

Linear Transformations

P 1. Determine if the transformation $T(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}) = \begin{bmatrix} 2x_1 \\ 4x_2 \\ 2x_3 \end{bmatrix}$ is linear? If the transformation is linear find the matrix representation of it.

P 2. Write down two methods of showing that a transformation is a linear transformation.

P 3. Use the theorem discussed to show that the following transformation $T(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}) = \begin{bmatrix} 2x_1 \\ 3x_2 \\ x_3 \end{bmatrix}$. is linear.

Their Inverse

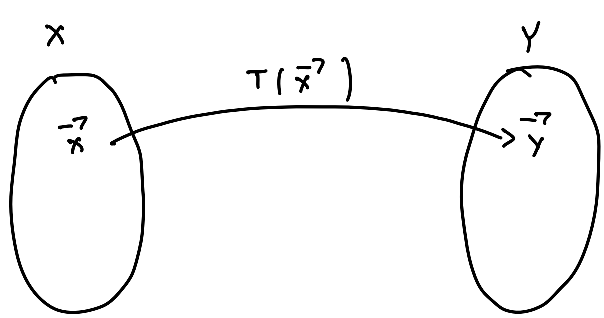
P 4. Find the inverse of the following matrix

$$\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}.$$

✓ left for monday ?

2.1

Recall T is a map from $X = \mathbb{R}^n = Y$.



Def: A transformation T is a function from $\mathbb{R}^m$ to $\mathbb{R}^n$. so $T(\vec{x}) = \vec{y}$ where $\vec{x} \in \mathbb{R}^m$ and $\vec{y} \in \mathbb{R}^n$.

Def: A transformation is a linear transformation LT from $\mathbb{R}^m$ to $\mathbb{R}^n$ if there exists an $n \times m$ matrix A such that $T(\vec{x}) = A\vec{x}$ for all $\vec{x} \in \mathbb{R}^m$.

Ex: Determine if the transformation is linear.

1) $T(\vec{x}) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ from $\mathbb{R}^2$ to $\mathbb{R}^2$.

No! Suppose $T(\vec{x}) = A\vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Then

$\exists\, \vec{a} \in \mathbb{R}^2$ such that $T(\vec{a}) \neq \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

$A\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\therefore$ A doesn't exist

2) $$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_2 \\ 2x_1 + 5x_2 \end{bmatrix}$$

Yes! we need an A such that

$$A\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + 3x_2 \\ 2x_1 + 5x_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$ $\therefore$ T is linear

We found the A!!!

Ex: Let $T(\vec{x}) = A\vec{x}$ where $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

Then

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} + 0\begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} + 0\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$$

Here $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is a standard vector denoted by $\vec{e}_1$.

$$T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$$

Thm: Let T be a LT. Then

$$A = \begin{bmatrix} | & | & & | \\ T(\vec{e}_1) & T(\vec{e}_2) & \cdots & T(\vec{e}_m) \\ | & | & & | \end{bmatrix}.$$

Ex:

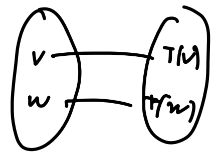
$$T(\vec{x}) = \begin{bmatrix} x_1 + 3x_2 \\ 2x_1 + 5x_2 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 + 0 \\ 2 + 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 + 3 \\ 0 + 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \quad \therefore \quad A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$$

Thm: Let T be a transformation from $\mathbb{R}^m$ to $\mathbb{R}^n$.

Then T is a LT if and only if,



$$\begin{cases} 1. & T(\vec{v}+\vec{w}) = T(\vec{v}) + T(\vec{w}) \\ 2. & kT(\vec{v}) = T(k\vec{v}) \end{cases}$$

OR $\left\{ 1. \quad T(k\vec{v}+\vec{w}) = kT(\vec{v}) + T(\vec{w}) \iff (1) \text{ \& } (2) \right.$

pf:

$$T(\vec{x}) = T\left(\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}\right)$$

$$= T(x_1\vec{e}_1 + x_2\vec{e}_2 + \cdots + x_n\vec{e}_n)$$

$$= T(x_1\vec{e}_1) + T(x_2\vec{e}_2) + \cdots + T(x_n\vec{e}_n) \quad \text{by (1)}$$

$$= x_1T(\vec{e}_1) + x_2T(\vec{e}_2) + \cdots + x_nT(\vec{e}_n) \quad \text{by (2)}$$

$$= \begin{bmatrix} | & | & & | \\ T(\vec{e}_1) & T(\vec{e}_2) & \cdots & T(\vec{e}_n) \\ | & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= A\vec{x} \quad \therefore \ T \text{ is linear}$$