Reading Questions 2

page 16 : 'Reduced Row-Echelon Form', 'Types of Elementary Row Operations'

1. Let $A = \begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix}$. Then $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is a column of A. F
2. The reduced row-echelon form of a matrix can contain fractions. T $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1/2 \\ 0 & 0 & 0 \end{bmatrix}$
3. How many types of elementary row operations are there?

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Section 1.2 Matrices and Vectors (Part 1)

Standard Representation

P 1. Consider the following matrix

$$A = \begin{bmatrix} 1 & 3 & 0 \\ 8 & 3 & 2 \\ 1 & 5 & 4 \end{bmatrix}.$$

1. List the rows and columns of A. List the diagonal entries of A.
2. What are the values for a_{13}, a_{32}, a_{23}?
3. Is A a square matrix?

P 2. Let

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \text{and} \quad \vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}.$$

The product of $\vec{x}$ and $\vec{y}$ in $\mathbb{R}^n$ is defined by

$$\vec{x} \cdot \vec{y} = x_1 y_1 + x_2 y_2 + \cdots + x_n y_n.$$

Use A from the previous problem to compute $\vec{x} \cdot \vec{y}$ for the following vectors.

1. $$\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \qquad \vec{y} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$
2. $$\vec{x} = \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix} \qquad \vec{y} = \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix}$$
3. The sum of $\vec{x}$ and $\vec{y}$ is defined to be $\begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_n + y_n \end{bmatrix}$ and for any real number c, $c\vec{x} = \begin{bmatrix} cx_1 \\ cx_2 \\ \vdots \\ cx_n \end{bmatrix}$.

   Show that $c(\vec{x} + \vec{y}) = c\vec{x} + c\vec{y}$.

Gauss-Jordan Elimination

P 3. Write the augment matrix for the following system of equations.

$$\left|\begin{array}{rcl} x_4 + 2x_5 - x_6 & = & 2 \\ x_1 + 2x_2 + x_5 - x_6 & = & 0 \\ x_1 + 2x_2 + 2x_3 - x_5 + x_6 & = & 2 \end{array}\right|$$

P 4. How many types of elementary rows operations can be performed on a matrix?

P 5. Put the following matrix in row reduced-echelon form and list the positions of the pivots.

$$\begin{bmatrix} 1 & 4 & 0 & 2 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 2 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

P 6. Write the general solution for the following augmented matrix.

$$\left[\begin{array}{cccc|c} 1 & 4 & 1 & 2 & 1 \\ 0 & 0 & 1 & 4 & 2 \\ 0 & 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

<u>Def:</u> A matrix is a rectangular array.

columns

<u>Ex:</u> $A = \begin{bmatrix} 3 & -3 \\ -6 & -1 \\ 2 & 8 \end{bmatrix}$ rows is a matrix.

A has 3 rows & 2 columns. Hence A is a 3×2 matrix.

rows columns

<u>Def:</u> A is square if A is $n \times n$.

<u>Ex:</u> $\begin{bmatrix} 1 & 8 \\ 0 & 2 \end{bmatrix} = B$ B is square.

$1 = b_{11}$ column row $b_{21} = 0$

$a_{11}, a_{22}, \cdots, a_{nn}$ are the diagonal entries of A.