WF 12-1

M 1

Reading Questions 1

page 1 : the entire page

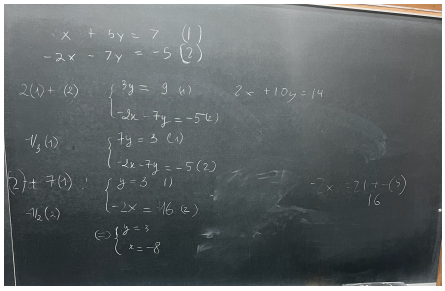
1. Algebra is the art of solving equations and system of equations. T
2. Linear algebra is the art of solving systems of linear equations. T
3. Solving systems of linear equations often arises in other areas of math, statistics, physics, astronomy, engineering, computer science, and economics. T
4. Write out a system of equations. x = 1, y = 2

Section 1.1 Introduction to Linear Systems (Part 1)

Linear Systems

P 1. Is the following set of equations a linear system?

$$1 = x + y \quad (1)$$
$$1 = 10x \quad (2)$$



P 2. Give an example of a set of equations which is not a linear system.

P 3. Find all solutions of the linear system by **eliminating variables**.

$$\left| \begin{array}{rcr} x + 5y & = & 7 \\ -2x - 7y & = & -5 \end{array} \right|$$

P 4. Find all solutions of the linear system by **eliminating variables**.

$$\left| \begin{array}{rcr} x + 2y + 3z & = & 6 \\ x + 1y + 2z & = & 6 \\ x + 2y + z & = & 4 \end{array} \right|$$

Geometric Interpretation

P 5. Use a graph to find the number of solutions to the following system of equations.

$$y + 2 = 20$$
$$y + x = 16$$

Def: A linear equation is an equation in which each term has at most one variable and degree ~~at most one~~ 0 or 1.

A linear system is a system/set of linear equations.

$x^{-1} = \frac{1}{x}$

Ex: a linear system

$$16 = x + y \quad (1)$$
$$100 = 5x + 10y \quad (2)$$

$x = 0 \quad y = 0$ is not a solution for the system. $16 \neq 0 + 0$

$x = 12 \quad y = 4$ is a solution for the system.

$16 = 12 + 4$ ✓

$100 = 5(12) + 10(4)$ ✓

$x = 12 \quad (1)$

$y = 4 \quad (2)$

Ex: Find the solutions to the following system of equations.

1
one
zero
infinite

$$x + 2y + 3z = 39 \quad (1)$$
$$x + 3y + 2z = 34 \quad (2)$$
$$3x + 2y + z = 26 \quad (3)$$

Eliminate the variables

$x = 4$

$y = 3$

$x - y = 4 - 3$

(2) - (1)

$$x + 2y + 3z = 39 \quad (1)$$

$$\boxed{x} + 3y + 2z = 34 \quad (2)$$

$$3x + 2y + z = 26 \quad (3)$$

(3) - 3(1)

$$x + 2y + 3z = 39 \quad (1)$$

$$y - z = -5 \quad (2)$$

$$\boxed{3x} + 2y + z = 26 \quad (3)$$

$$x + \boxed{2y} + 3z = 39 \quad (1)$$

$$y - z = -5 \quad (2)$$

$$\boxed{-4y} - 8z = -91 \quad (3) \qquad 26 - 3(39)$$

(1) - 2(2)

(3) + 4(2)

$$x + 5z = 49 \quad (1)$$

$$y + -z = -5 \quad (2)$$

$$\boxed{-12}z = -111 \quad (3)$$

$-\frac{1}{12}(3)$

$$x + \boxed{5z} = 49 \quad (1)$$

$$y + \boxed{-z} = -5 \quad (2)$$

$$z = \frac{111}{12} \quad (3)$$

(1) - 5(3)

(2) + (3)

$$x = 49 - 5\left(\frac{111}{12}\right) \quad (1)$$

$$y = -5 + \left(\frac{111}{12}\right) \quad (2)$$

$$z = \frac{111}{12} \quad (3)$$

How many solutions does a system of equations have?

1, 0, ∞

Ex:

$x = 1$

$y = 2$

1 sol

$y + x = 2$

$y + x = 3$

0 sols

$y + x = 2$

$0 = 1$

$y + x = 3$

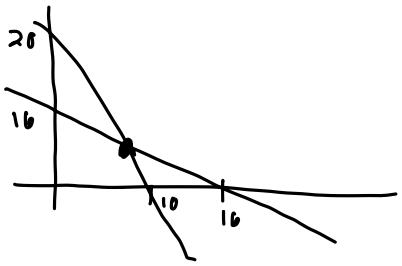
$0 = 0$

∞ sols

Ex:

$y + x = 16$ a line

$5y + 10x = 100$ a line



a solution occurs at the intersection

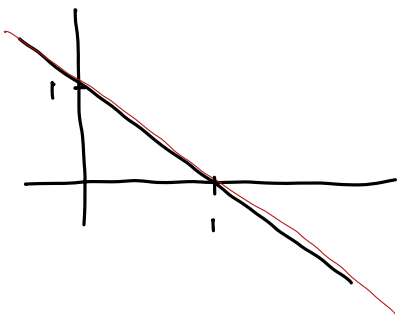
(4, 12)

one solution

Ex:

$x + y = 1$

$2x + 2y = 2$

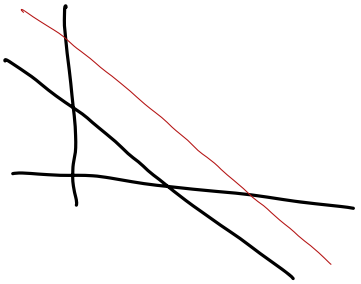


infinitely many solutions

Ex:

$x + y = 1$

$x + y = 3$



no solution