P 1. Compute $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. In general, does $AB = BA$?

P 2. Let the matrix representation of the linear transformations T and S be

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 2 & 2 & 1 \\ 3 & 2 & 1 & 1 \end{bmatrix}$$

respectively. Find the matrix representation of $T \circ S$.

P 3. Compute the following product of matrices $\begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$.

P 4. Find a 3×3 matrix A which is not I_3 or $-I_3$ such that $AA = I_3$.

P 5. Find a 2×2 matrix A such that $A^2 \neq I$ and $A^4 = I$.

P 6. Let P be the matrix projection onto $\vec{u} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$. Is there a matrix Q such that $QP = I$?

P 7. Let $A = \begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$. Suppose $\vec{b} \in \mathbb{R}^2$. How many solutions does the linear equation $A\vec{x} = \vec{b}$ have?

P 8. Find the vectors that span the image of $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$.

P 9. Find a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ whose image is the line spanned by the vector $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$.

P 10. Find the vectors that span the kernel of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

P 11. Show that the line $x_1 + x_2 = 0$ in $\mathbb{R}^2$ is a subspace of $\mathbb{R}^2$.

P 12. Determine if the following set is a subspace of $\mathbb{R}^3$. Be sure to justify your answer.

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x + y + z = 1 \right\}$$

P 13. Let T be a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$. Show that the kernel of T is a subspace of $\mathbb{R}^n$.

P 14. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 3 & 6 \end{bmatrix}$.

1. Find a basis for the kernel.
2. Find a basis for the image.
3. Determine the dimensions for each of the previously found subspaces.
4. Use the dimension of the image of A to determine the number of free variables for the system $A\vec{x} = \vec{0}$.
5. Use the dimension of the kernel of A to determine the rank of A.

P 15. Find $[\vec{x}]_{\mathfrak{B}}$ where $\mathfrak{B} = \{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}5\\6\end{bmatrix}\}$ and $\vec{x} = \begin{bmatrix}-4\\4\end{bmatrix}$.

P 16. Find the $\mathfrak{B}$-matrix for the linear transformation $T(\vec{x}) = A\vec{x}$, where

$$A = \begin{bmatrix}0 & 1\\2 & 3\end{bmatrix}$$

and $\mathfrak{B} = \{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}\}$.

P 17. Compute the determinant for the matrix M.

$$M = \begin{bmatrix}0 & 0 & 1 & 0 & 2\\5 & 4 & 3 & 2 & 1\\1 & 3 & 5 & 0 & 7\\2 & 0 & 4 & 0 & 6\\0 & 0 & 3 & 0 & 4\end{bmatrix}$$

P 18. Let $A = \begin{bmatrix}1 & 2 & 0\\2 & 2 & 1\\0 & 2 & 1\end{bmatrix}$. Compute $\det(A^T)$.

P 19. Given some numbers a, b, c, d, e and f such that

$$\det\begin{bmatrix}a & 1 & d\\b & 1 & e\\c & 1 & f\end{bmatrix} = 7,$$

find

$$\det\begin{bmatrix}a & 3 & d\\b & 3 & e\\c & 3 & f\end{bmatrix}.$$

P 20. Let $A = \begin{bmatrix}1 & 3 & 0 & 3\\0 & 3 & 0 & 3\\0 & 0 & 3 & 3\\0 & 0 & 0 & 2\end{bmatrix}$. Suppose A and B are similar matrices. Find $\det(B)$ and $\det(A^{-1})$.

P 21. If $\begin{bmatrix}1\\-1\\-1\end{bmatrix}$ is an eigenvector of the matrix $\begin{bmatrix}4 & 1 & 1\\-5 & 0 & -3\\-1 & -1 & 2\end{bmatrix}$ what is its eigenvalue?

P 22. Let $\vec{v}$ be an eigenvector for A. Is $\vec{v}$ an eigenvector for A^3? If so what are the eigenvalues?

P 23. Let $\vec{v}$ be an eigenvector for A. Is $\vec{v}$ an eigenvector for $A + \sigma I$? If so what are the eigenvalues?

P 24. Let A be an $n \times n$ matrix.

1. Write down the characteristic equation of A.
2. Write down the characteristic polynomial of A where $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigenvalues of A.

P 25. Find the eigenvalues for the matrix

$$\begin{bmatrix}-3 & 0 & 0 & 0 & 0 & 0\\0 & 2 & 0 & 0 & 0 & 0\\0 & 0 & 1 & 0 & 0 & 0\\0 & 0 & 0 & 3 & 0 & 0\\0 & 0 & 0 & 0 & 1 & 1\\0 & 0 & 0 & 0 & 1 & 2\end{bmatrix}$$

and their algebraic multiplicities.

P 26. Find the eigenvectors for the matrix $A = \begin{bmatrix} -3 & 0 & 4 \\ 0 & -1 & 0 \\ -2 & 7 & 3 \end{bmatrix}$.

P 27. Give two different examples of a distribution vector.

P 28. Give an example of a positive transition matrix.

P 29. Let $A = \begin{bmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{bmatrix}$.

1. Use the example in today's lecture to find the closed formula for A^t where t is an arbitrary positive integer.
2. Find $\lim_{t\to\infty} A^t$.
3. Compute $\lim_{t\to\infty} A^t \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$.