Section 1.1 Introduction to Linear Systems (Part 1)

P 1. Find all solutions of the linear system by **eliminating variables**.

$$\left|\begin{array}{rcr} x+5y & = & 7 \\ -2x-7y & = & -5 \end{array}\right|$$

P 2. Find all solutions of the linear system by **eliminating variables**.

$$\left|\begin{array}{rcr} x+2y+3z & = & 6 \\ x+1y+2z & = & 6 \\ x+2y+z & = & 4 \end{array}\right|$$

P 3. Use a graph to find the number of solutions to the following system of equations.

$$y+2=20$$
$$y+x=16$$

Section 1.2 Matrices and Vectors (Part 1)

P 4. Consider the following matrix

$$A=\begin{bmatrix} 1 & 3 & 0 \\ 8 & 3 & 2 \\ 1 & 5 & 4 \end{bmatrix}.$$

1. List the rows and columns of A. List the diagonal entries of A.
2. What are the values for a_{13}, a_{32}, a_{23}?
3. Is A a square matrix?

P 5. Let

$$\vec{x}=\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \text{and} \quad \vec{y}=\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}.$$

1. Compute $\vec{x}\cdot\vec{y}$ for the following vector.

$$\vec{x}=\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \qquad \vec{y}=\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

2. Show that $c(\vec{x}+\vec{y})=c\vec{x}+c\vec{y}$.

P 6. Write the augment matrix for the following system of equations.

$$\left|\begin{array}{rcr} x_4+2x_5-x_6 & = & 2 \\ x_1+2x_2+x_5-x_6 & = & 0 \\ x_1+2x_2+2x_3-x_5+x_6 & = & 2 \end{array}\right|$$

P 7. Put the following matrix in row reduced-echelon form and list the positions of the pivots.

$$\begin{bmatrix} 1 & 4 & 0 & 2 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 2 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

P 8. Write the general solution for the following augmented matrix.

$$\begin{bmatrix} 1 & 4 & 1 & 2 & | & 1 \\ 0 & 0 & 1 & 4 & | & 2 \\ 0 & 0 & 1 & 4 & | & 2 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$$

Section 1.3 On the Solutions of Linear Systems (Part 1)

P 9. For each of the following augmented matrices write its solutions and state the number of solution it has.

$$A = \begin{bmatrix} 1 & 0 & | & 5 \\ 0 & 1 & | & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 2 & 1 & | & 0 \\ 0 & 1 & 3 & 1 & | & 0 \\ 0 & 1 & 3 & 1 & | & 0 \\ 0 & 1 & 3 & 1 & | & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 3 \end{bmatrix}$$

P 10. For each of the following matrices write the rref and determine its rank.

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$$

P 11. Suppose that A is an $n \times n$ coefficient matrix and the rank of A is n. How many solutions does the system of equations have? Justify your answer.

P 12. Compute

$$\begin{bmatrix} 1 & 2 & 2 \\ 3 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 2 \\ 3 & 2 & 3 \\ 2 & 3 & 2 \end{bmatrix}, \quad 4 \begin{bmatrix} 1 & 2 & 2 \\ 3 & 2 & 3 \\ 2 & 3 & 2 \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 & 2 \\ 3 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}.$$

P 13. Compute the product $A\vec{x}$ by using the rows of A.

$$A = \begin{bmatrix} 4 & 4 & 2 \\ 5 & 5 & 1 \\ 3 & 3 & 1 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

P 14. Let $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 3 & 2 \\ 1 & 3 & 1 \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$. Write $\begin{bmatrix} 13 \\ 15 \\ 12 \end{bmatrix}$ as a linear combination of the columns of A.

Section 2.1 Linear Transformations and Their Inverse (Part 1)

P 15. Determine if the transformation $T(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}) = \begin{bmatrix} 2x_1 \\ 4x_2 \\ 2x_3 \end{bmatrix}$ is linear? If the transformation is linear find the matrix representation of it.

Section 2.2 Linear Transformations in Geometry (Part 1)

P 16. Find the matrix corresponding to the transformation $T(\vec{x}) = 2023\vec{x}$. How does this transformation transform the vector $\vec{x}$?

P 17. Let L be the line in $\mathbb{R}^2$ that consists of all scalar multiples of the vector $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Find the orthogonal projection of the vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ onto L.

P 18. Find the matrix for the linear transformation that reflects vectors in $\mathbb{R}^2$ over the line $y = -2x$.