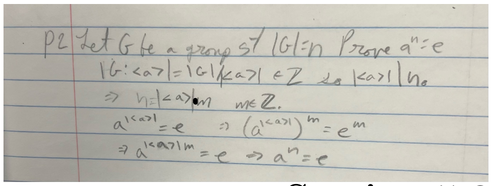
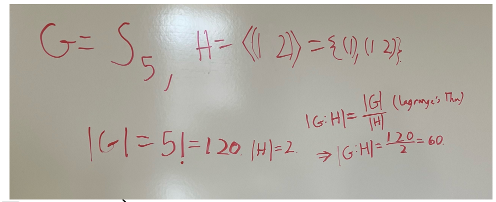




Section 5.2 Lagrange Theorem (Part 1)

Lagrange Results

$|G:\langle a\rangle| = \dfrac{|G|}{|\langle a\rangle|} \in \mathbb{Z}$

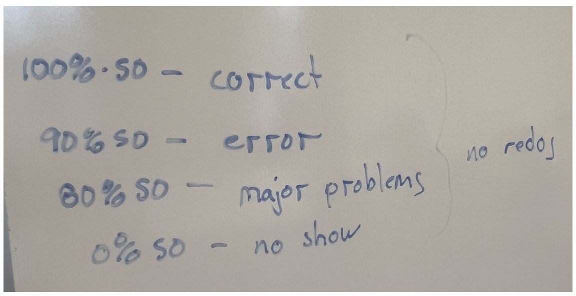
P 1. Let $G = S_5$ and $H = <(12)>$. What is $|G:H|$?

P 2. Let G be a group such that $|G| = n$. Prove $a^n = e$. $a \in G$

P 3. Let G be a finite group such that $H \leq K \leq G$. Prove $|G:K|\cdot|K:H| = |G:H|$.

Let G be a finite group such that $H, K \leq G$. Then

Presentation $|G:K| = |H : H\cap K|$ iff $G = HK$



less than 8 mins

use math notation instead of full sentences

HW 9 - redos ✓

5.2 Lagrange's Theorem

<u>lem:</u> Let G be a group such that $H \leq G$ and $g \in G$.

Then $|H| = |Hg|$

<u>pf:</u> Define $\sigma : H \to Hg$ where $\sigma(h) = hg$.

Then σ is clearly well defined.

Suppose $\sigma(h_1) = \sigma(h_2)$. Then $h_1 g = h_2 g \Rightarrow h_1 = h_2$.

So σ is 1-1.

Suppose $y \in Hg$. Then $y = hg$ for some $h \in H$.

$\sigma(h) = y$. $\therefore$ σ is a 1-1 correspondance.

Thm: Let G be a finite group such that $H \leq G$.

Then $|G:H| = \frac{|G|}{|H|}$ or $|G| = |G:H|\,|H|$.

pf: We know G/H partition G. Hence

$$|G| = |Hg_1| + |Hg_2| + \cdots + |Hg_{|G:H|}|$$

$$= \underbrace{|H| + |H| + \cdots + |H|}_{|G:H| \text{ times}} \quad \text{by lem}$$

$$= |G:H|\,|H|.$$

cor: Let G be a finite group such that $a \in G$.

Then $o(a)$ divides $|G|$.

pf: Let $H = \langle a \rangle$. By Lagrange thm

$\frac{|G|}{|H|} = |G:H| \in \mathbb{Z}^{\text{or } \mathbb{N}}$. Hence $|H|$ divides $|G|$.

Since $o(a) = |H|$ it follows that $o(a)$ divides $|G|$.

cor: Let G be a group such that $|G|$ is prime and

$e \neq x \in G$. Then $G \cong \mathbb{Z}_p$ p-prime.

pf: WTS G is cyclic.

Consider $\langle x \rangle$. By Lagrange's Thm

$\frac{|G|}{|\langle x \rangle|} \in \mathbb{Z}$. Since $|G|$ is prime and $x \neq e$

it follows that $|\langle x \rangle| = |G|$.

$\therefore \quad \sigma(x) = 1$ defines an isomorphism from G to $\mathbb{Z}_p$.

$|G| = 5 \qquad G \cong \mathbb{Z}_5 \qquad G = \langle x \rangle \qquad \mathbb{Z}_5 = \langle 1 \rangle$

$x \to 1$

$x^2 \to 2$

$x^3 \to 3$

$x^4 \to 4$

$e \to 0$