

- P 1.** List all 3 cycles in S_4 .
- P 2.** Prove the following statement. Let n be a positive integer. If σ and τ are disjoint cycles in S_n then $\sigma\tau = \tau\sigma$.
- P 3.** Write $(123)(24)(321)$ as a product of disjoint cycles.
- P 4.** Write $(1234)(231)$ as a product of transpositions.
- P 5.** What is the order of $(123)(25)(46)$ in S_7 ?
- P 6.** Let $\sigma, \tau \in S_n$. Prove or disprove. If σ and τ are both odd then $\sigma\tau$ is even.
- P 7.** List the elements of A_4 .
- P 8.** Let $\Omega = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}\}$. Let $\sigma = (123)$. Then S_4 acts on Ω where $\sigma \cdot \{a, b\} = \{\sigma(a), \sigma(b)\}$. Compute $\sigma \cdot \{1, 4\}$ and $\sigma \cdot \{2, 3\}$ where $\sigma = (123)$.
- P 9.** Find a subgroup of S_4 which is isomorphic to Z_4 . Hint Z_4 acts on $\{0, 1, 2, 3\}$ where $g \cdot a = g + a \pmod{4}$.
- P 10.** Let $G = GL(n, \mathbb{R})$ and let Ω be the set of all real $n \times n$ matrices. Let $A \in G$ and $B \in \Omega$. Define $A \cdot B = BAB^{-1}$. Show that G acts on Ω .
- P 11.** Let G be a group such that $H \leq G$. Prove or disprove: H acts on G where $h \cdot g = gh^{-1}$.
- P 12.** Let $G = Z_6$. Let $H = \langle 3 \rangle$ and $g = 2$. Write the elements of gHg^{-1} .
- P 13.** Let D_8 act on $\{1, 2, 3, 4\}$. Let $S = \{a, ab\}$. Draw the Cayley graph.
- P 14.** Let D_8 act on $\{1, 2, 3, 4\}$. Find $\text{Stab}_{D_8}(3)$.
- P 15.** Let S_4 act on $\{1, 2, 3, 4\}$ defined by the action $\sigma \cdot a = \sigma(a)$. Find $\text{Stab}_{S_4}(2)$.
- P 16.** Let $a \sim b$ if $a, b \in \mathbb{Z}$ and $a \leq b$. Find $\text{cl}(2)$.
- P 17.** Let $G = S_7$. Let $H = \langle (23), (132) \rangle$ act on $\Omega = [7]$ where $h \cdot a = h(a)$ for $h \in H$ and $a \in \Omega$. What are the orbits of Ω ?
- P 18.** What are the conjugacy classes of S_4 ?
- P 19.** Let $(1432), (1324) \in S_4$. Find $\sigma \in S_4$ such that $(1432) = \sigma(1324)\sigma^{-1}$.
- P 20.** Let $G = S_4$ and $H = \langle (123) \rangle$. List the right cosets of H in G .
- P 21.** Let $G = S_5$ and $H = \langle (12) \rangle$. What is $|G : H|$?