P 1. Find the vectors that span the image of $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$.

P 2. Find the vectors that span the kernel of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

P 3. Show that the line $x_1 + x_2 = 0$ in $\mathbb{R}^2$ is a subspace of $\mathbb{R}^2$.

P 4. Let T be a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$. Show that the kernel of T is a subspace of $\mathbb{R}^n$.

P 5. Find the $\mathfrak{B}$-matrix for the linear transformation $T(\vec{x}) = A\vec{x}$, where

$$A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$$

and $\mathfrak{B} = \{\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\}$.

P 6. Suppose $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} = S \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix} S^{-1}$. Find a matrix C which is similar to $\begin{bmatrix} 9 & 8 \\ 16 & 17 \end{bmatrix}$.

P 7. Compute the determinant for the matrix M.

$$M = \begin{bmatrix} 0 & 0 & 1 & 0 & 2 \\ 5 & 4 & 3 & 2 & 1 \\ 1 & 3 & 5 & 0 & 7 \\ 2 & 0 & 4 & 0 & 6 \\ 0 & 0 & 3 & 0 & 4 \end{bmatrix}$$

P 8. Find $\det(A^2)$ for $A = \begin{bmatrix} 1 & 81 & 80 & 88 \\ 0 & 2 & 86 & 84 \\ 0 & 0 & 3 & 87 \\ 0 & 0 & 0 & 4 \end{bmatrix}$

P 9. Let $A = \begin{bmatrix} 1 & 4 & 6 & 8 \\ 1 & 2 & 3 & 8 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 6 & 8 \end{bmatrix}$. Compute the determinant of A using elementary row operations and determine if A is invertible.

P 10. Does there exist an invertible matrix S and a diagonal matrix B such that $AS = SB$ where A is the linear transformation which rotates a vector 180° in $\mathbb{R}^2$? Explain your answer.

P 11. Let $\vec{v}$ be an eigenvector for A. Is $\vec{v}$ an eigenvector for $A + \sigma I$? If so what are the eigenvalues?

P 12. Find the eigenvalues for the matrix $\begin{bmatrix} -3 & 0 & 4 \\ 0 & -1 & 0 \\ -2 & 7 & 3 \end{bmatrix}$.

P 13. Find the eigenvectors for the matrix $A = \begin{bmatrix} -3 & 0 & 4 \\ 0 & -1 & 0 \\ -2 & 7 & 3 \end{bmatrix}$.

P 14. For each eigenvalue λ of a find the algebraic and geometric multiplicity of λ.

P 15. Suppose $\vec{x}, \vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ are distribution vectors. Let the columns of A are the vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$. Is $A\vec{x}$ a distribution vector. Explain your answer.

P 16. Let $A = \begin{bmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{bmatrix}$.

1. Find $\lim_{t \to \infty} A^t$.

2. Compute $\lim_{t \to \infty} A^t \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$.

P 17. Find orthonormal vectors $\vec{u}_1$ and $\vec{u}_2$ in the subspace $V = \text{span}(\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \\ -8 \end{bmatrix})$ such that $V = \text{span}(\vec{u}_1, \vec{u}_2)$. Check your answer.

P 18. Find a 2×2 matrix R such that $\begin{bmatrix} 2 & 2 \\ 1 & 7 \\ -2 & -8 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2 & -2 \\ 1 & 2 \\ -2 & -1 \end{bmatrix} R$. Check your answer.