

Reading Questions 9

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1. The binary relation \leq is the relation $\{(a, b) | a, b \in \mathbb{R} \text{ and } a \leq b\}$.
2. A relation R is transitive if $(a, b), (b, c) \in R$ then $(a, c) \in R$.
3. What is the additive inverse of 5?

Section 4.1 The Division Algorithm (Part 1)

Well-Ordering Principle

P 1. Which of the following sets contains a least element?

- (a) \mathbb{N} (b) \mathbb{Z} (c) $(2, 8]$

The Division Algorithm

P 2. Find integers q and r such that $51 = 7q + r$ where $0 \leq r < 7$.

P 3. The Division Algorithm: Let $a, b \in \mathbb{Z}, b \neq 0$. Prove there exist unique integers q and r , with $0 \leq r < |b|$ such that $a = qb + r$.

Hint: Try using proof by cases.

- Case 1: $a = 0, b \neq 0$.
- Case 2: $a, b > 0$.
- Case 3: $a > 0, b < 0$.
- Case 4: $a < 0, b > 0$.
- Case 5: $a < 0, b < 0$.

P 4. Find integers q and r , with $0 \leq r < 20$ such that $-3,315 = 20q + r$.

P 5. Find integers q and r , with $0 \leq r < 20$ such that $3,315 = -20q + r$.

P 6. Using the division algorithm, show that if $x \in \mathbb{Z}$ then $x = 2k$ for some integer k or $x = 2k + 1$ for some integer k .