Reading Questions 7

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1. The inverse of a function is a set.
2. Let f be a function from the set A to the set B. Then the range of the inverse of f is A.
3. The composition of two functions is a set.

Section 3.2 Inverse and Composition (Part 1)

Inverses

P 1. Find the inverse relation of the following relation.

$$R = \{(1,8),(3,3),(4,3),(2,1),(5,2)\}$$

P 2. Determine if the following functions from the set $\{1,2,3,4,5\}$ itself have an inverse. If so find the inverse.

$$f = \{(1,3),(3,4),(4,3),(2,1),(5,2)\} \quad \text{and} \quad g = \{(1,2),(3,1),(2,4),(4,3),(5,5)\}$$

Compositions

P 3. Let $A = \{1,2,3,4\}, B = \{a,b,c,d\}$ and $C = \{r,s,t,u,v\}$ and define the functions $f : A \to B$ and $g : B \to C$ by

$$f = \{(1,b),(2,d),(3,a),(4,a)\} \text{ and } g = \{(a,u),(b,r),(c,r),(d,s)\}.$$

Determine $g \circ f$ and $(g \circ f)(1)$.

Theorem

The functions $f : A \to B$ and $g : B \to A$ are inverses of each other if and only if

$$(g \circ f)(a) = a \text{ and } (f \circ g)(b) = b$$

for all $a \in A$ and for all $b \in B$.

P 4. Let $f : A \to A$ and $g : A \to A$ be functions. Show that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

P 5. Let $f : A \to B$ and $g : B \to C$ be functions. Prove that if $g \circ f$ is one-to-one and f is onto then g is one-to-one.