Reading Questions 16

page 291 Definition 9.2.4

page 291 Example 4

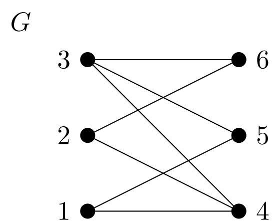
1. A bipartite graph is a graph with no edges. F
2. A complete bipartite graph is a graph with all possible edges. F
3. Suppose $V(G) = \{1, 2, 3, 4\}$ and $E(G) = \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 1\}\}$. Give a bipartition sets V_1 and V_2 for the graph G.

   $\{1,3\}$ $\{2,4\}$

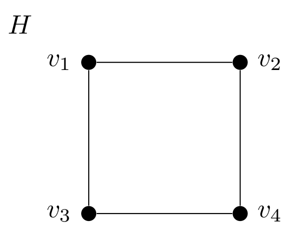
Section 9.2 Definitions and Basic Properties (Part 1)

Definitions and Basic Properties

P 1. Compute $V(G)$ and $E(G)$ for the following graph.



P 2. Compute $\sum_{v \in V(H)} \deg v$ for the following graph. Also compute $|E(H)|$.



P 3. What is the maximum number of edges in a graph containing n vertices?

P 4. Prove that at any party an even number of people must have shaken an odd number of hands.

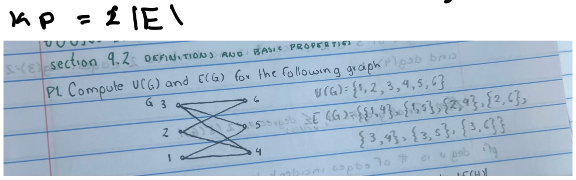
deg v

$\Sigma \deg v = 2|E|$

P 5. Let G be a graph and $p = 2k + 1$ for some integer k. Prove that if $\deg v = p$ for all $v \in V(G)$ then $p \mid |E(G)|$.

$\Sigma p = 2|E|$ G is regular

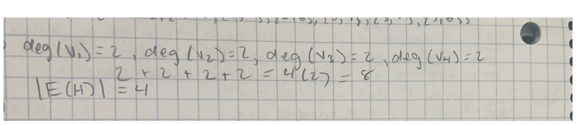
$kp = 2|E|$


section 9.2 DEFINITIONS AND BASIC PROPERTIES

P1. Compute V(G) and E(G) for the following graph

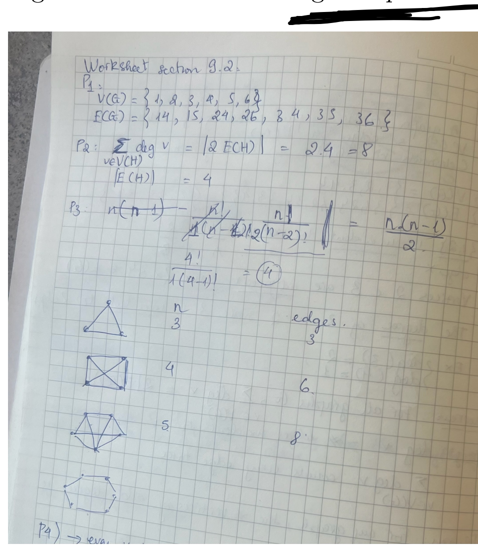
V(G) = {1, 2, 3, 4, 5, 6}

E(G) = {{1,4}, {1,5}, {2,4}, {2,6}, {3,4}, {3,5}, {3,6}}


deg(v1) = 2, deg(v2) = 2, deg(v3) = 2, deg(v4) = 2

2 + 2 + 2 + 2 = 4(2) = 8

|E(H)| = 4


Worksheet section 9.2

P1:

V(G) = {1, 2, 3, 4, 5, 6}

E(G) = {14, 15, 24, 26, 34, 35, 36}

P2: $\sum_{v \in V(H)} \deg v = |2 E(H)| = 2 \cdot 4 = 8$

|E(H)| = 4

P3: $\frac{n!}{2(n-2)!} = \frac{n(n-1)}{2}$

$\frac{4!}{1(4-1)!} = 4$

n	edges
3	3
4	6
5	8

P4) → even

9.2

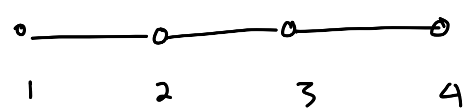
Def: A graph G is a pair consisting of a vertex set $V(G)$ and an edge set $E(G)$ (whose elements are subsets of $V(G)$ of size 2 called edges).

Ex: The graph G has a vertex set

$$V(G) = \{1, 2, 3, 4\}$$ and an edge set

$$E(G) = \{\{1,2\}, \{2,3\}, \{3,4\}\} = \{12, 23, 34\}$$

G



edge $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ vrtx

$\{1,2\}$ is <u>incident</u> to 1 and 2

vertices 2 and 3 are <u>adjacent</u> since $\{2,3\} \in E(G)$

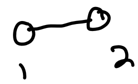
the <u>degree</u> of 3 is 2 since 3 is incident to 2 edges. $\deg(3) = 2$ and $\deg(4) = 1$

Thm: For all graphs G, $\displaystyle\sum_{v \in V(G)} \deg v = 2|E(G)|$.

sketch pf:

$\deg v$ is # of edges incident to v

$\sum_{v \in V(G)} \deg v$ counts every edge twice.



$\deg(1)$ counts $\{1,2\}$

$\deg(2)$ counts $\{1,2\}$

Thm: For any graph the number of vertices of odd degree is even.

pf: Let A be the set of vertices of odd degree.

Let $B = V(G) \setminus A$. $V(G) = A \dot{\cup} B$

$$\sum_{v \in V(G)} \deg(v) = \sum_{v_i \in A} \deg v_i + \sum_{v_j \in B} \deg v_j$$

$$= \underbrace{\sum_{v_i \in A} 2k_i + 1 + \underbrace{\sum_{v_j \in B} 2k_j}_{\text{even}}}_{\text{even by the previous thm}}$$

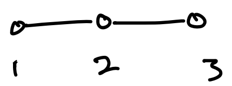
$\deg v_i$ is odd

$\Rightarrow \exists k_i \in \mathbb{Z}$ s.t. $\deg(v_i) = 2k_i + 1$

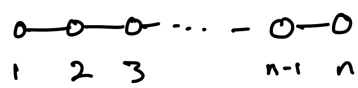
$\Rightarrow \sum_{v_i \in A} 2k_i + 1$ is even

Therefore $|A|$ is even.

path

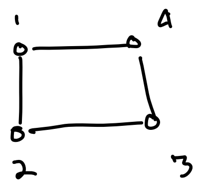


P_3

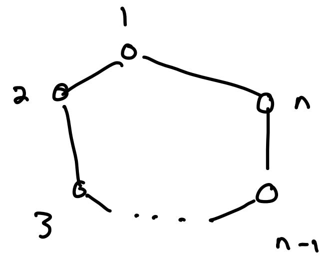


P_n

cycle

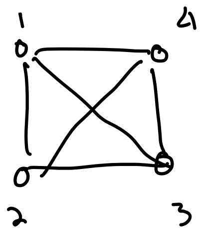


C_4



C_n

complete graph

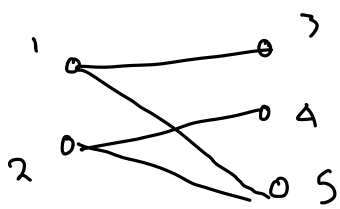


K_4

a graph containing all possible edges.

K_n

bipartite



Here

$A = \{1, 2\}$

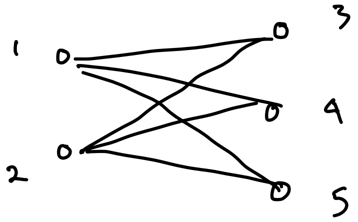
$B = \{3, 4, 5\}$

$V(G) = A \cup B$

$A \cap B = \emptyset$

$E(A) = E(B) = \emptyset$

complete bipartite



$K_{2,3} = K_{3,2}$

a bipartite graph which contains all possible edges

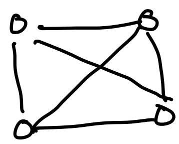
$|K_{n,m}| = ?$

Def: The graph H is a subgraph of G if

$V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.

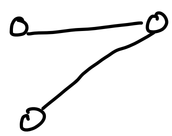
Ex: Let

G



then

H



is a subgraph of G.