Reading Questions 15

page 285 Problem 1

1. The problem was clearly stated.
2. You understood the solution.
3. What are your plans for Thanksgiving break?

Section 9.1 A Gentle Introduction (Part 1)

Applications

P 1. Create a pseudograph which contains 6 edges and 8 vertices.

P 2. Find the Instant Insanity solution for the following cubes. (A solution exists.)

	W		
R	W	B	G
	W		

Cube 1

	R		
G	W	B	B
	W		

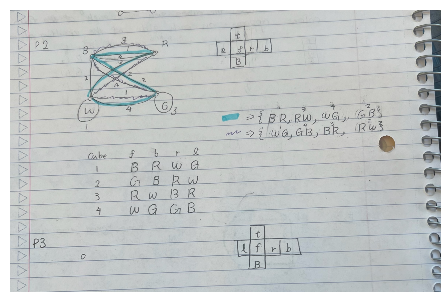
Cube 2

	R		
B	G	R	G
	W		

Cube 3

	G		
B	B	R	G
	W		

Cube 4



P 3. Find the Instant Insanity solution for the following cubes.

	R		
R	W	G	W
	B		

Cube 1

	B		
R	R	B	W
	G		

Cube 2

	W		
G	G	R	B
	B		

Cube 3

	R		
W	B	R	G
	G		

Cube 4

P 4. Acquaintance Relation Problem Does every set of six people contain three mutual acquaintances or three mutual strangers?
How can we use pseudographs to model the acquaintance problem? Defined $V(\mathcal{G})$ and $E(\mathcal{G})$.
Form a question related to the pseudographs whose answer solves the acquaintance problem.
Answer the question.

6.3

P 3. There are 15 students in a discrete math class. Show that there are at least 2 students that have birthdays in the same month. In this problem what are the Pigeons and what are the holes?

15 students - pigeons

12 months - holes

by the pigeon hole prin. we have at least 2 students with the same b day month

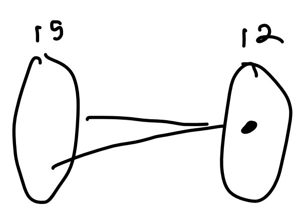
P 4. Show that among $n + 1$ arbitrarily chosen integers, there must exist two whose difference is divisible by n.

division alg

$a_1, \dots, a_{n+1}$

$a_i = q_t n + r_t$

$\omega = r_t - r_j$



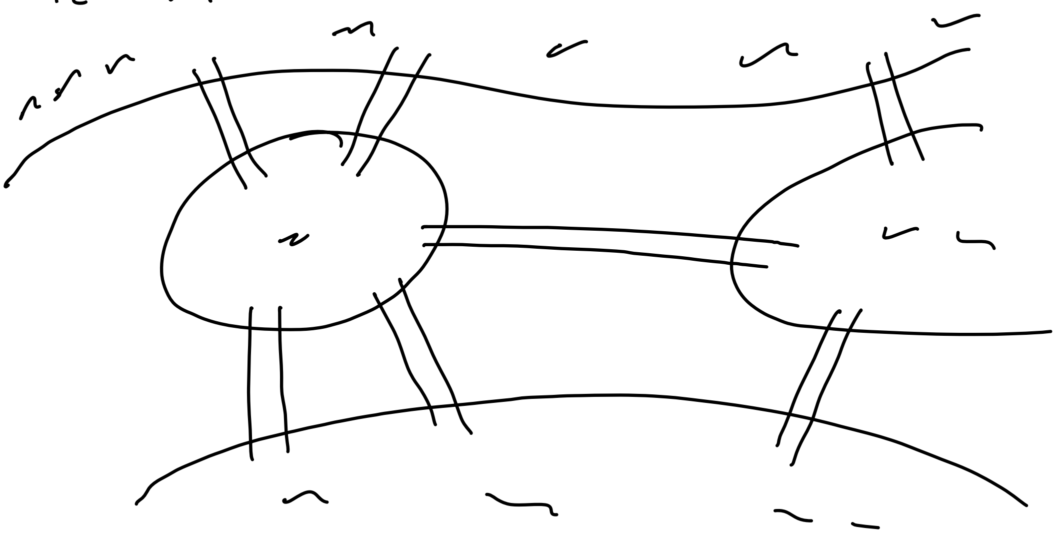
$i \in \{1, \dots, n+1\}$ - pigeons

$r_t \in \{0, \dots, n-1\}$ - holes

9.1

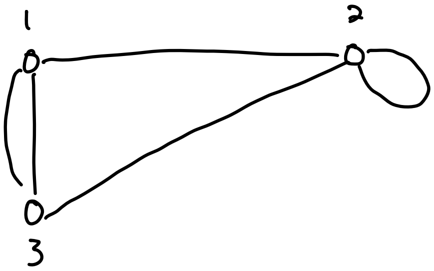
The Konigsberg Problem

The city of Konigsberg was located on the Pregal river in Prussia. The city occupied two islands plus areas on both banks. The regions were linked by seven bridges. The citzens wondered whether they could leave home, cross every bridge exactly once and then return.



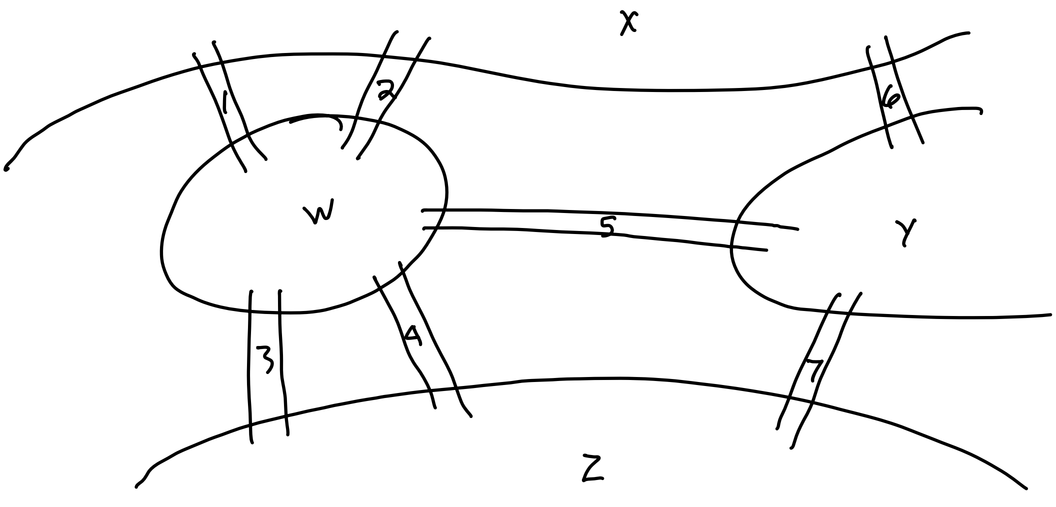
Def: A pseudograph G is a pair consisting of a vertex set $V(G)$ (whose elements are vertices) and a multiset $E(G)$ (whose elements are multisets of size 2 which contain elements of $V(G)$).

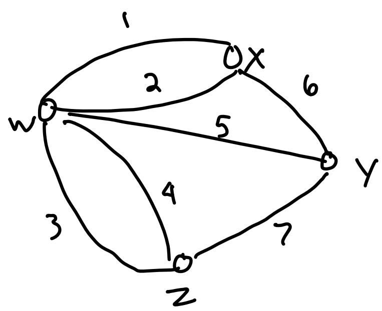
Ex: Let G be the following graph



multi edges $\qquad\qquad$ loop

$V(G) = \{1, 2, 3\}$ $\qquad$ $E(G) = \{\{1,3\}, \{1,3\}, \{1,2\}, \{2,2\}, \{2,3\}\}$



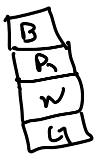


Can we travel along every edge of G and return to the original vertex? No, every vertex must be used an odd number of times.

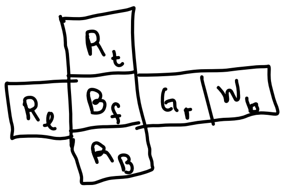
Instant Insanity

Suppose we are given four cubes, each of whose six faces colored (R) red, (B) blue, (G) green, or (W) white.

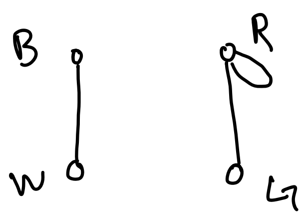
Can we stack the cubes such that each of the four colors appear on each side of the resulting colum?



A Cube



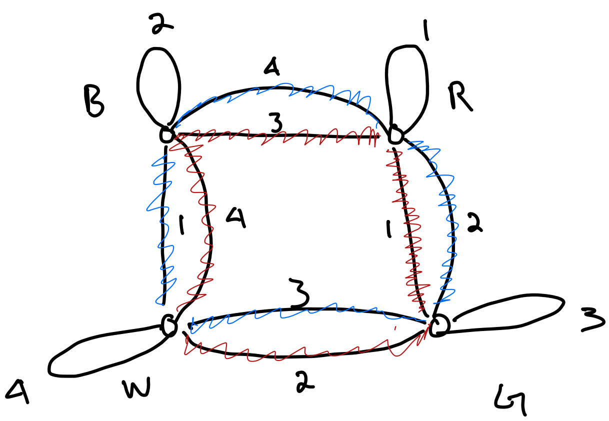
A graph



A solution

cube	f	b	r	l
1	B	W	R	G
2	W	G	G	W
3	G	R	B	R
4	R	B	W	B

Ex:



some 4 cubes

Does there exist two distinct sets of cycles (use all vertices) such that all edges are different colors?

$\Rightarrow \{ \overset{1}{BW}, \overset{3}{WG}, \overset{2}{GR}, \overset{4}{RB} \}$ - 2 columns

$\Rightarrow \{ \overset{2}{WG}, \overset{1}{GR}, \overset{3}{RB}, \overset{4}{BW} \}$ - 2 columns

cube	f	b	r	l
1	B	W	G	R
2	G	R	W	G
3	W	G	R	B
4	R	B	B	W