Reading Questions 14

page 200 Principle 6.3.1

page 200 Problem 11

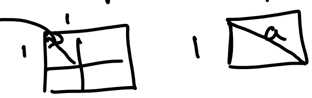
1. If n objects are put into m boxes and $n > m$, then at least one box contains at least 2 objects.

T

2. In problem 11, the squares are the objects and the points are the boxes.

F $1^2 + 1^2 = a^2$

3. In problem 11, why is the length of the diagonal equal to $\sqrt{2}$?



Section 6.3 The Pigeonhole Principle (Part 1)

More Counting Problems

Section 6.2

P 1. How many three-digit numbers contain the digits 2 and 5? Hint : Add and Mult Rule

P 2. Let $A = \{1, 2, \ldots, n\}$ and $B = \{1, 2\}$. Show that there are 2^n functions from A to B.

Existence Problems

Section 6.3

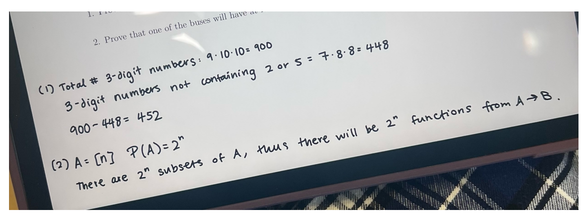
P 3. There are 15 students in a discrete math class. Show that there are at least 2 students that have birthdays in the same month. In this problem what are the Pigeons and what are the holes?

P 4. Show that among $n + 1$ arbitrarily chosen integers, there must exist two whose difference is divisible by n.

P 5. Thirty buses are to be used to transport 2000 students from South Hadley to Amherst. Each bus has 80 seats. Assume one seat per passenger.

1. Prove that one of the buses will carry at least 67 passengers.

2. Prove that one of the buses will have at least 14 empty seats.



2 5 x   2 x 5   x 2 5
5 2 x   5 x 2   x 5 2
2|A| 2|B| 2|C|

2 5 2   2 2 5   2 2 5
2 5 5   5 2 2   5 5 2

2 2 2
5 5 5   2(|A| + |B| + |C| + 4)

6.2 $|A_1 \times A_2 \times \cdots \times A_n| = |A_1| \cdot |A_2| \cdot \cdots \cdot |A_n|$

Ex: How many ways can a password of length 3 be created using only 2 letters a and b.

Addition Rule

_ _ _

fix

a a a

a a b

a b a

a b b

fix

b a a $\leftrightarrow$ (b, a, a)

b a b

b b a

b b b

Mult. Rule

$|\{a,b\} \times \{a,b\} \times \{a,b\}| = |\{a,b\}| \cdot |\{a,b\}| \cdot |\{a,b\}|$

$= 2 \cdot 2 \cdot 2 = 8$

Thm: The number of ways to create a password of length k using n letters is n^k.

$|A| = n$

$|A_1 \times A_2 \times \cdots \times A_k| = |A_1| \cdot |A_2| \cdot \cdots \cdot |A_k|$

$= \underbrace{n \cdot n \cdot \cdots \cdot n}_{k\text{-times}} = n^k$

Thm: The number of elements in the power set of $[n]$ is 2^n, where $n \geq 1$. $|\mathcal{P}([n])| = 2^n$

$\{1, 3, 5\} \subseteq [5]$

$\updownarrow$

10101

$11011 \leftrightarrow \{1, 2, 4, 5\}$

$|\{0,1\}^n| = 2^n$

6.3

Ten people attend a party to socialize. Given that each person did not shake their own hand, and shook another person's hand at most once, is it the case that at least two people shook the same number of hands.

n_i = # of hands person i shook

Assume no two people shook the same # of hands

$i \in \{1, \dots, 10\}$

$n_i \in \{0, 1, 2, \dots, 9\}$

<u>case 1</u> Assume $n_1 = 0$. Then $n_i \neq 9$.

WLOG

<u>$n_1 = 0$, $n_2 = 1$, $n_3 = 2$,</u> $\dots$, $n_{10} \in \{0, \dots, 8\}$.

$\exists\, n_i$ s.t $n_{10} = n_i$ where $i \in \{1, \dots, 9\}$

<u>case 2</u> Assume $n_i \neq 0$ $\forall i$

WLOG

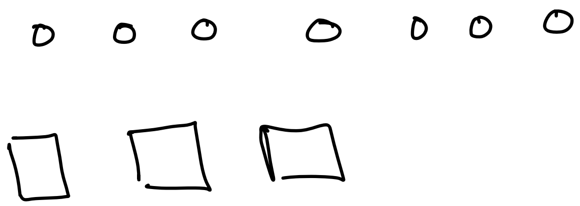
$n_1 = 1$, $n_2 = 2$, $\dots$, $n_{10} \in \{1, \dots, 9\}$

$\exists\, n_i$ s.t $n_{10} = n_i$ where $i \in \{1, \dots, 9\}$.

<u>Pigeonhole Principle</u>

If n pigeons are put into m holes with $n > m$

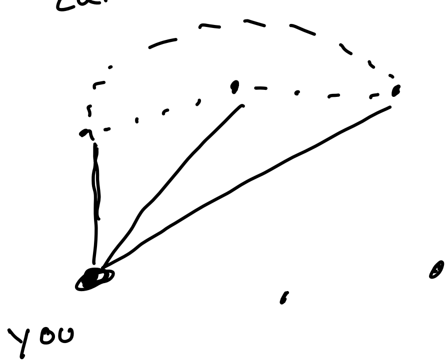
then there is a least 1 hole containing at least $\lceil \frac{n}{m} \rceil$ pigeons.



<u>Ex:</u> In any group of six people, at least three must be mutual friends or at least three must be mutual strangers.

By the pigeon hole P you know 3 people or you don't know 3 people.

case 1



case 2