Reading Questions 12

$|A \cup B| = |A| + |B| - |A \cap B|$

page 185 Proposition 6.1.1 (a) and its proof

1. If A and B are finite sets then $|A \cup B| = |A| + |B|$. F
2. You understood the proof of Proposition 6.1.1 part (a). T
3. Suppose $|A| = 4$, $|B| = 3$, and A and B have no common elements. What is $|A \cup B|$?

   " 7

Section 6.1 The Principle of Inclusion-Exclusion (Part 1)

The Cardinality of the Union of two Sets

In a group of 15 pizza experts, ten like bacon, seven like mushrooms, and six like both. How many people liked at least one topping?

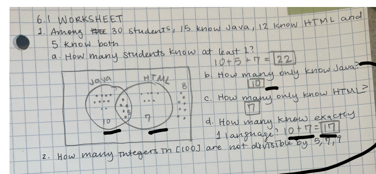
P 1. Among the 30 students registered for a course in discrete mathmematics, 15 people know the JAVA programming language, 12 know HTML, and 5 know both of these languages.

(a) How many students know at least on of JAVA or HTML?

(b) How many students know only JAVA?

(c) How many know only HTML?

(d) How many know exactly one of the languages JAVA and HTML?


6.1 WORKSHEET
1. Among ~~the~~ 30 students, 15 know Java, 12 know HTML and 5 know both
a. How many students know at least 1?
$10 + 5 + 7 = \boxed{22}$
b. How many only know Java?
$\boxed{10}$
c. How many only know HTML?
$\boxed{7}$
d. How many know exactly 1 language? $10 + 7 = \boxed{17}$
2. How many integers in [100] are not divisible by 5, 7, 9

$|A \cup B| = |A| + |B| - |A \cap B|$

A Generalization

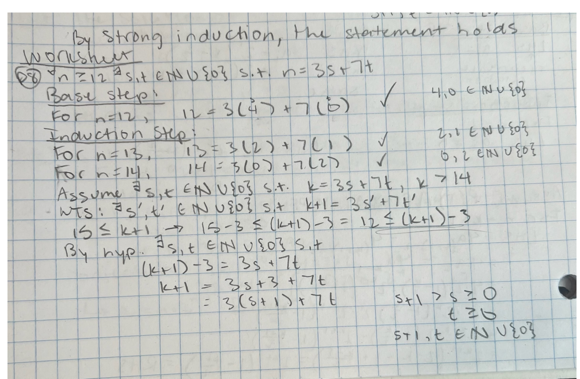
> **Theorem**
>
> Let a and n be integers such that $0 < a < n$. Then the number of positive integers which are divisible by a and less than n is $\lfloor \frac{n}{a} \rfloor$.

" $\{1, 2, 3, \ldots, 100\}$

P 2. How many integers in [100] are not divisible by 5, 7, or 9?

P 3. How many integers between 1 and 500 are divisible by 3 but not by 5 or 6?

P 4. How many integers between 1 and 10,000 are divisible by 3 and 7 but not by either 5 or 11?


By strong induction, the statement holds
Worksheet
(28) $\forall n \geq 12\ \exists s, t \in \mathbb{N} \cup \{0\}$ s.t. $n = 3s + 7t$
Base step:
For $n = 12$, $12 = 3(4) + 7(0)$ ✓ $4, 0 \in \mathbb{N} \cup \{0\}$
Induction Step:
For $n = 13$, $13 = 3(2) + 7(1)$ ✓ $2, 1 \in \mathbb{N} \cup \{0\}$
For $n = 14$, $14 = 3(0) + 7(2)$ ✓ $0, 2 \in \mathbb{N} \cup \{0\}$
Assume $\exists s, t \in \mathbb{N} \cup \{0\}$ s.t. $k = 3s + 7t$, $k > 14$
WTS: $\exists s', t' \in \mathbb{N} \cup \{0\}$ s.t $k+1 = 3s' + 7t'$
$15 \leq k+1 \rightarrow 15 - 3 \leq (k+1) - 3 = 12 \leq (k+1) - 3$
By hyp. $\exists s, t \in \mathbb{N} \cup \{0\}$ s.t
$(k+1) - 3 = 3s + 7t$
$k + 1 = 3s + 3 + 7t$
$= 3(s+1) + 7t$
$s + 1 > s \geq 0$
$t \geq 0$
$s+1, t \in \mathbb{N} \cup \{0\}$

P 8. Prove $^\forall n \geq 12\ ^\exists s, t \in \mathbb{N} \cup \{0\}$ such that $n = 3s + 7t$.

Ex: Prove $\forall n \in \mathbb{N}$, $n \geq 8$ $\exists s, t \in \mathbb{N} \cup \{0\}$ such that

$n = 3s + 5t$.

pf:

Base step:

For $n = 8$, $8 = 3(\underset{s}{\underset{\|}{1}}) + 5(\underset{t}{\underset{\|}{1}})$ ✓ $1, 1 \in \mathbb{N} \cup \{0\}$

Ind step:

For $n = 9$, $9 = 3(\underset{s}{\underset{\|}{3}}) + 5(\underset{t}{\underset{\|}{0}})$ ✓ $0, 3 \in \mathbb{N} \cup \{0\}$

For $n = 10$, $10 = 3(\underset{s}{\underset{\|}{0}}) + 5(\underset{t}{\underset{\|}{2}})$ $0, 2 \in \mathbb{N} \cup \{0\}$

Assume $\exists s, t \in \mathbb{N} \cup \{0\}$ s.t. $K = 3s + 5t$, $K > 10$.

WTS $\exists s', t' \in \mathbb{N} \cup \{0\}$ s.t $K+1 = 3s' + 5t'$.

$11 \leq K+1 \Rightarrow 8 = 11 - 3 \leq (K+1) - 3$

By hyp $\exists s, t \in \mathbb{N} \cup \{0\}$ such that

$\underline{(K+1) - 3} = 3s + 5t$

$K+1 = 3s + 3 + 5t$

$= 3(s+1) + 5t$ $s+1 > s \geq 0$, $t \geq 0$ $s+1, t \in \mathbb{N} \cup \{0\}$

By strong ind the statement holds.

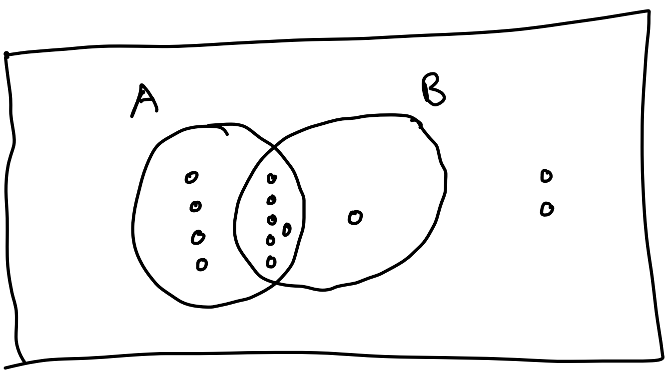
6.1

Def: A multiset is a set that may contain multiple copies of the same element.

Ex: $\{1,2,2,3\}$ is a multiset

Let A be the people who like bacon

Let B be the people who like mushroom

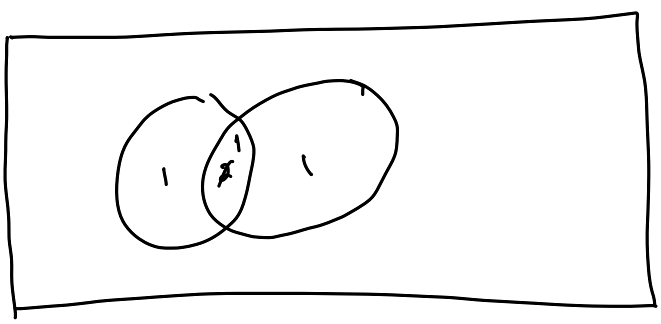


$A = \{a,b,c\}$

$B = \{1,b,2\}$

$|A \cup B| = |A| + |B| - |A \cap B|$

$M = \{a,b,c,1,b,2\}$



$|A| = 10$ $\quad |B| = 7$ $\quad |A \cap B| = 6$

$|A \cup B| = |A| + |B| - |A \cap B|$

$= 10 + 7 - 6 = 11$