Reading Questions 11

page 162 Definition 5.2.1

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page 164 Definition 5.2.3

1. The sequence $-17, -12, -7, -1, 3, \ldots$ is an arithmetic sequence. F
2. The sequence $2, 6, 18, 54, \ldots$ is a geometric sequence. T $r=3$ $2 \cdot 3^i$
3. The first term in the Fibonacci sequence is 0. F
4. Is the sequence $1, 1, 1, \ldots$ an arithmetic sequence or geometric sequence or both?

both $r = 1$ $d = 0$

Section 5.2 Recursively Defined Sequences (Part 1)

Recurrence Relation

P 1. Write out the first 6 terms of the sequence $f : \mathbb{N} \to \mathbb{Z}$ such that $f : a \mapsto 3a$.

P 2. Is the equation $a_k = 2$ where $k \geq 1$ a recurrence relation? If so, write out the sequence and the $5th$ term of the sequence.

P 3. What is the solution to the recurrence relation $a_{k+1} = 3a_k$ where $k \geq 0$ and $a_0 = 1$.

P 4. Let $a_1 = 1$ and $a_n = 2a_{n-1} + 1$ for $n \geq 2$. Write out the first 6 terms. Conjecture a solution for the recurrence relation.

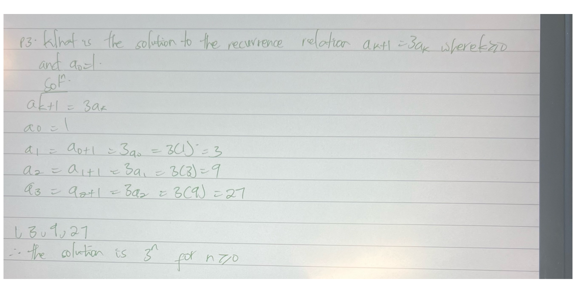
P 5. Let $a_1 = 1, a_2 = 3$ and $a_n = 2a_{n-1} - a_{n-2}$ for $n \geq 3$. Write out the first 6 terms. Conjecture a solution for the recurrence relation.

Strong Principle of Mathematical Induction

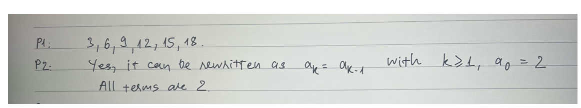
P 6. Let $a_1 = 1, a_2 = 3$ and $a_n = 2a_{n-1} - a_{n-2}$ for $n \geq 3$. Conjecture a solution for the recurrence relation and prove it using Strong Mathematical Induction.

P 7. Let $a_0 = 1$, $a_1 = 4$, and $a_n = 4a_{n-1} - 4a_{n-2}$ for $n \geq 2$. Prove that $a_n = 2^n(n+1)$ for all $n \geq 0$.

P 8. Prove $^\forall n \geq 12$ $^\exists s, t \in \mathbb{N} \cup \{0\}$ such that $n = 3s + 7t$.


P3: What is the solution to the recurrence relation $a_{k+1} = 3a_k$ where $k \geq 0$ and $a_0 = 1$.
Sol^n:
$a_{k+1} = 3a_k$
$a_0 = 1$
$a_1 = a_{0+1} = 3a_0 = 3(1) = 3$
$a_2 = a_{1+1} = 3a_1 = 3(3) = 9$
$a_3 = a_{2+1} = 3a_2 = 3(9) = 27$

1, 3, 9, 27
$\therefore$ the solution is 3^n for $n \geq 0$


P1: 3, 6, 9, 12, 15, 18.
P2: Yes, it can be rewritten as $a_k = a_{k-1}$ with $k \geq 1$, $a_0 = 2$
All terms are 2.

5.1

Def: A sequence is a function (or infinite list of elements) whose domain is some subset of integers and range is a set of elements (real numbers).

Ex: $2, 4, 6, 8, \ldots$ is a sequence

$f : \mathbb{N} \to \mathbb{Z}$ s.t. $f : n \mapsto \underbrace{2n}_{n\text{th term}}$

Ex: $1, 4, 9, 16, 25, \ldots$ has the n^{th} term of

$n \geq 1$, n^2

$n = 6 \Rightarrow 6^2$ - 6^{th} term of the sequence

Def: A recurrence relation is an equation where the nth term of the sequence is expressed in terms of the other terms of the sequence.

Ex: The equation $a_k = 2a_{k-1}$ is a recurrence relation where $a_0 = 1$ and $k \geq 1$.

$1, 2, 4, 8, 16, \ldots$

Ex: $a_k = 2a_{k-1} + 2a_{k-2}$ for $k \geq 2$

$a_0 = 1 \quad a_1 = 2$

Def: The solution of the recurrence relation is the n^{th} term of the sequence.

Ex: $a_0 = 1 \quad a_n = 2a_{k-1} \quad k \geq 1$

$1, 2, 4, 8, 16, \ldots$

$a_n = 2^n \quad n \geq 0$

Here is a solution for the recurrence relation.

Find the solution for the following recurrence relation.

Ex: $a_0 = 1 \quad a_1 = 4 \quad a_n = 4a_{n-1} - 4a_{n-2}$ for $k \geq 2$.

$a_0 = 1 \quad a_1 = 4 \quad a_2 = 4 \cdot 4 - 4 \cdot 1 = 12$

$a_3 = 4(12 - 4) = 32 \quad a_4 = 4(32 - 12) = 80$

$1, 4, 12, 32, 80, 192, \ldots$

conjecture $1 = 1 \cdot 2^0 \quad 4 = 2 \cdot 2^1 \quad 12 = 3 \cdot 2^2$

$32 = 4 \cdot 2^3 \quad 80 = 5 \cdot 2^4$

$a_n = n \cdot 2^{n-1}$ for $n \geq 1$.

Strong Induction

Let $P(n)$ be a statement concerning the integers $\{n_0, n_1, \ldots\}$. Suppose

(1) $P(n_0)$ is true, an

(2) $P(n_0) \wedge P(n_1) \wedge \cdots \wedge P(n_k) \Rightarrow P(n_{k+1})$

for all $k \geq 0$.

Then $P(n)$ is true for all elements in $\{n_0, n_1, \ldots\}$.

Ex: Let $a_1 = 1$, $a_2 = 0$, and $a_n = 4a_{n-1} - 4a_{n-2}$ for $n \geq 3$. Prove $a_n = 2^n(1 - \frac{n}{2})$ for $n \geq 1$.

pf: by strong induction

Base step for $n = 1$

LHS $a_1 = 1$

recurrence relation

RHS $a_1 = 2^1(1 - \frac{1}{2})$

n-th

$= 2 \cdot \frac{1}{2} = 1$

Ind step

For $n = 2$

LHS $a_2 = 0$

RHS $a_2 = 2^2 \cdot (1 - \frac{2}{2})$

$= 4 \cdot 0 = 0$

Assume $P(1) \wedge P(2) \wedge \cdots \wedge P(k)$.

For $n = k+1$.

$$LHS = 4a_{(k+1)-1} - 4a_{(k+1)-2}$$

$$= 4a_k - 4a_{k-1}$$

$$= 4\left[2^k\left(1 - \frac{k}{2}\right) - 2^{k-1}\left(1 - \frac{k-1}{2}\right)\right]$$

$$= 2^2 \cdot 2^k\left(1 - \frac{k}{2}\right) - 2^2 \cdot 2^{k-1}\left(1 - \frac{k-1}{2}\right)$$

$$= 2^{k+1} \cdot 2\left(1 - \frac{k}{2}\right) - 2^{k+1}\left(1 - \frac{k-1}{2}\right)$$

$$= 2^{k+1}\left(2 - k - 1 + \frac{k-1}{2}\right)$$

$$= 2^{k+1}\left(1 - \frac{k+1}{2}\right) = RHS$$