Reading Questions 8

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1. A one-to-one correspondence is a relation. T
2. There is a one-to-one correspondence between the sets $\{a, b, x\}$ and $\{1, 2, 3, 4\}$ F
3. The sets $\{x \in \mathbb{R} | x^2 + 1 = 0\}$ and $\{\}$ have the same cardinality. T
4. What is the set $2\mathbb{Z}$? $\ldots, -4, -2, 0, 2, 4, \ldots$

Section 3.3 One-to-One Correspondence and the Cardinality of Sets (Part 1)

One-to-One Correspondence

P 1. Is there a one-to-one correspondence from $\underbrace{|\{1, 2, 3, \ldots, n\}|}_{=n}$ to the $\underbrace{|\text{empty set}|}_{=0}$? no ~~Is~~ Are the sets finite or infinite?

P 2. Show that the set $\{x, y, z, 1\}$ is a finite set.

$x \to 1$
$y \to 2$
$z \to 3$
$1 \to 4$

$1 \to 1$
$2 \to 2$
$\vdots$
$n \to n$

$\underbrace{A \xrightarrow{f} B \xrightarrow{g} C}_{g \cdot f}$

Countable Sets

P 3. Show that the set ~~$\mathbb{N} \cup \{0\}$~~ is a countable set. $\mathbb{N} \setminus \{1\} \to \mathbb{N}$ $f(x) = x - 1$

P 4. Determine if the set $\{3^n | n \in \mathbb{Z}\}$ is countable. ($\mathbb{Z}$ → W)

$W \to \mathbb{Z} \to \mathbb{N}$
$f(3^i) = i$

P 5. It is true that every infinite set contains a countable set. Use this fact to prove the following theorem.
Theorem: Let S be an infinite set and let x be an element not in S. Prove that $|S| = |S \cup \{x\}|$.

$$P = \{x_1, x_2, x_3, \ldots\} \subseteq S$$

$$f(x) = \begin{cases} x & \text{if } x \notin P \\ x_{i-1} & \text{if } x \in P \end{cases}$$

3.3

Def: A one-to-one correspondence is a function that is both 1-1 and onto.

Def: The cardinality of a set A, denoted by $|A|$, is the number of elements in A. Sets A and B have the same cardinality if the exists a 1-1 correspondence between A and B. $|A| = |B|$

Ex: Let $A = \{1, 2\}$ and $B = \{a, b\}$. Then

$f = \{(1, a), (2, b)\}$ is a 1-1 correspondence from A to B. Hence $|A| = |B|$.

Def: The set A is a finite set if A is the empty set or there is a 1-1 correspondence from A to $\{1, 2, \ldots, n\}$ for some positive integer n. Otherwise A is an infinite set.

Ex: The set $A = \{a, b, c\}$ is finite as

$a \to 1$
$b \to 2$
$c \to 3$

is a 1-1 correspondence.

The set $\mathbb{N}$ is infinite. $|\{1, \ldots, n\}| = n$

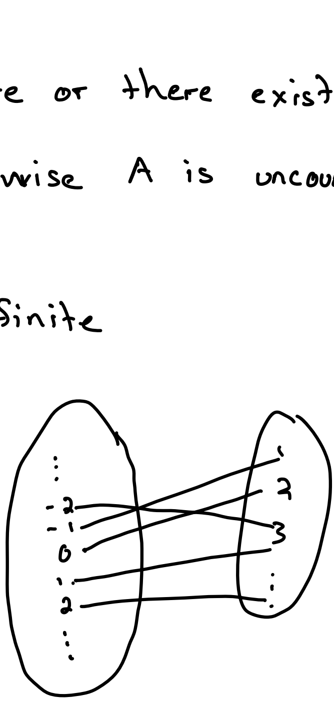
$|\mathbb{N}| \neq n$

Def: A set A is countable if A is finite or there exists a 1-1 correspondence from A to $\mathbb{N}$. Otherwise A is uncountable.

Ex: $A = \{1, 2, 3, 4\}$ is countable as A is finite

$\mathbb{Z}$ is countable $f : \mathbb{Z} \to \mathbb{N}$

$$f(a) = \begin{cases} 2a+2 & \text{if } a \geq 0 \\ 2|a| - 1 & \text{if } a < 0 \end{cases}$$



Ex: $A = \{x_1, x_2, x_3, x_4, \dots\}$ is countable

and

$\{x_2, x_4, x_6, \dots\} \subseteq A$ and countable

$f: \mathbb{N} \cup \{0\} \to \mathbb{N}$

$f: x \mapsto x+1$

Ex: $\mathbb{N} \cup \{0\}$ is countable

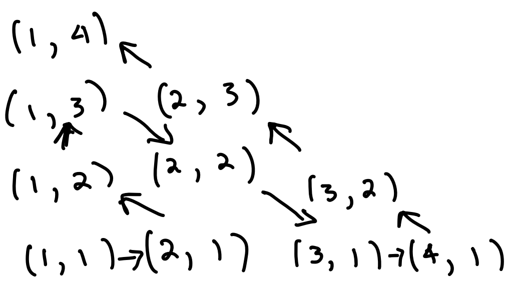
Let S be a countable set. Then

$S \cup \{x\}$ is countable where x is anything.

Thm: Let X, Y, Z be countable sets. Then

$$|(X \times Y) \times Z| = |X \times (Y \times Z)|$$

Ex: $\mathbb{N} \times \mathbb{N}$ is countable.



$f: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Q}$

$f(m, n) = \frac{m}{n}$