Reading Questions 6

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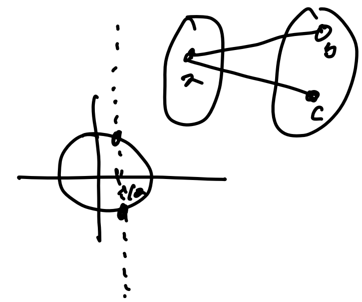
1. A function is a set. T
2. Let f be a function from the set A to the set B. Then the range of f is B. F
3. All functions are one-to-one. F
4. What does it mean for a function to be a one-to-one?



Section 3.1 Basic Terminology (Part 1)

Functions as Sets

P 1. Write a binary relation from the set $\{1, 2, 3\}$ to the set $\{1, 2, 3\}$. $\{\ \}$

P 2. Is the set $\{(1, 2), (3, 1), (2, 1)\}$ a function from the set $\{1, 2, 3\}$ to the set $\{1, 2, 3\}$. Yes!

P 3. Let $A = \{1, 2, 3\}$ and $B = \{a, b, c, d\}$. Give an example of a relation from A to B containing exactly three elements such that the relation is not a function from A to B. $\{(1,a),(2,b),(2,c)\}$

P 4. Let $A = \{a, b, c, d\}$ and $B = \{x, y, z\}$. Then $f\{(a, y), (b, z), (c, y), (d, z)\}$ is a function from A to B. Determine $\operatorname{dom} f$ and $\operatorname{rng} f$.

P 5. Let $A = \{w, x, y, z\}$ and $B = \{r, s, t\}$. Give an example of a function $f : A \to B$ that is neither one-to-one nor onto.

Functions as Sets Proofs

P 6. Show that the function $f = \{(x_1, x_2)\,|x_1^2 = x_2\}$ from $\mathbb{N}$ to $\mathbb{N}$ is one-to-one. Is the function onto?

P 7. Let $f = \{(x, y)|y = 3x + 5\}$ be the function from $\mathbb{N}$ to $\mathbb{N}$. Show that f a bijective.

P 8. Determine the values for $|\frac{\pi}{-3}|$, $\lfloor \pi \rfloor$, $\lceil e \rceil$ and $\lfloor \pi + e \rfloor$.

3.1 Functions

Def: Let A and B be sets. A binary relation from A to B is a subset of $A \times B$.

Ex: Let $A = \{1, 2, 3\}$ and $B = \{4, 5\ 6\}$. Then

$\{(1,5), (1,4), (2,6)\}$ is a binary relation from A to B.

Def: A function (map) from A to B is a binary relation from A to B, called f, such that

$\forall a \in A$ there is exactly one $b \in B$ such that $(a,b) \in f$.

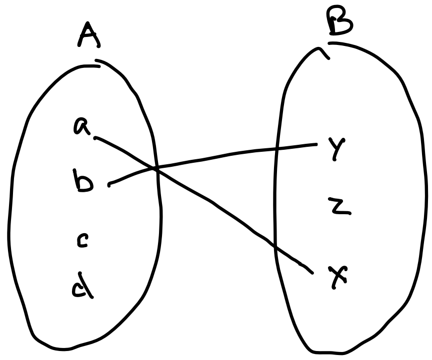
$\forall a \in A\ \exists!\, b \in B$ s.t. $(a,b) \in f$

Notation:

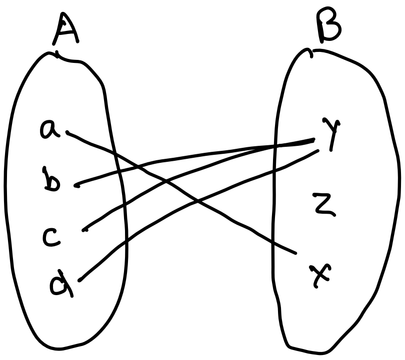
$f : A \to B$

$(a,b) \in f \Rightarrow f : a \mapsto b \Rightarrow f(a) = b$

$R = \{(a,x), (b,y)\}$



$f = \{(a,x), (b,y), (c,y), (d,y)\}$



Def: Let f be a function from A to B

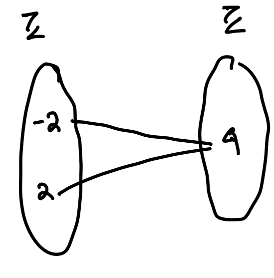
1. domain of f is A, $\operatorname{dom} f$
2. the target space of f is B
3. range of f is $\{ b \in B \mid \exists a \in A, (a,b) \in f \}$, $\overset{\text{im } f}{\overset{\|}{\operatorname{rng} f}}$
4. f is one-to-one if $x_1 \neq x_2$ then $(x_1, b) \neq (x_2, b)$

   if $(x_1, b) = (x_2, b)$ then $x_1 = x_2$
5. f is onto if $\operatorname{rng} f = B$
6. f is bijective (or a bijection) if f is 1-1 and onto

Ex: Let $f = \{ (x_1, x_2) \mid x_1, x_2 \in \mathbb{Z},\ x_1^2 = x_2 \}$

$\overset{\text{function}}{f} : \overset{\text{domain}}{\mathbb{Z}} \to \mathbb{Z} \leftarrow$ target space

$\operatorname{dom} f = \mathbb{Z}$ $\quad$ $\operatorname{rng} f = \{ x \in \mathbb{Z} \mid \sqrt{x} \in \mathbb{Z} \}$



$(2,4) \in f$ as $2^2 = 4$ and $(-2, 4) \in f$

Hence f is not 1-1.

$\nexists x \in \mathbb{Z}$ s.t $(x, 2) \in f$. $\therefore$ f is not onto.

Thm: Let $f: \mathbb{Z} \to \mathbb{Z}$ such that $f: x \mapsto 3x^3 - x$.

1. The function is 1-1.
2. f is not bijective.

pf: (1) Proof by Contradiction. WTS $(x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2))$

Assume $x_1, x_2 \in \mathbb{Z}$ and $x_1 \neq x_2$. Suppose $f(x_1) = f(x_2)$.

Then $3x_1^3 - x_1 = 3x_2^3 - x_2$

$\Rightarrow \quad 3x_1^3 - 3x_2^3 = x_1 - x_2$

$\Rightarrow \quad 3(x_1^3 - x_2^3) = x_1 - x_2$

$\Rightarrow \quad 3(x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2) = x_1 - x_2$

$\Rightarrow \quad x_1^2 + x_1x_2 + x_2^2 = \frac{1}{3}$ since $x_1 \neq x_2$

Since $x_1, x_2 \in \mathbb{Z}$ $x_1^2 + x_1x_2 + x_2^2 \in \mathbb{Z} \Rightarrow \Leftarrow$.

$\therefore$ f is 1-1