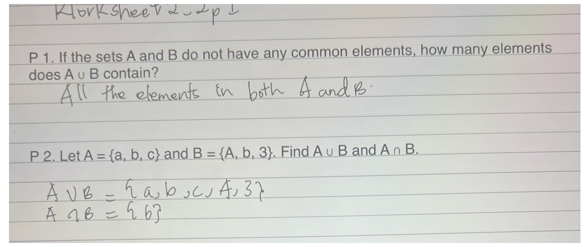
Reading Questions 5



Worksheet 2.2 p1

P 1. If the sets A and B do not have any common elements, how many elements does A ∪ B contain?

All the elements in both A and B.

P 2. Let A = {a, b, c} and B = {A, b, 3}. Find A ∪ B and A ∩ B.

$A \cup B = \{a, b, c, A, 3\}$
$A \cap B = \{b\}$

page 46 Problem 9

page 48 Example 12

1. The complement of a set is a set. T
2. Let A and B be sets. Then $A \times B = B \times A$. F
3. The complement of the set $\{1, 2, 3\}$ is the set $\mathbb{Z} \setminus \{1, 2, 3\}$. F

Section 2.2 Operations on Sets (Part 1)

Notation

P 1. If the sets A and B do not have any common elements, how many elements does $A \cup B$ contain?

P 2. Let $A = \{a, b, c\}$ and $B = \{A, b, 3\}$. Find $A \cup B$ and $A \cap B$.

P 3. What is the complement of the set $\{1, 3, 9, 27\}$ with respect to the set

$$\{1, 2, 3, 4, 8, 9, 16, 27, 32, 81\}?$$

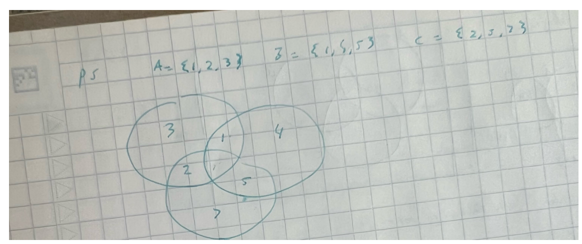
P 4. What is the complement of the set of rational number with respect to the set of real numbers?

Venn Diagram

P 5. Make a Venn diagram for the sets $A = \{1, 2, 3\}$, $B = \{1, 4, 5\}$, and $C = \{2, 5, 7\}$.

P 6. Let $A = \{1, 2\}$ and $B = \{x, y, z\}$. Find $B \times A$ and B^2.

P 7. Prove that for any sets A and B, $(A \cap B)^c = A^c \cup B^c$.



2.2

Def: The union of two sets A and B, denoted by $A \cup B$, is the set containing all elements from A and B. The intersection of two sets A and B, denoted by $A \cap B$, is the set containing all elements in both A and B.

Ex: $A = \{1, 2, 3\}$ $\{4, 5, \pi, 2, 3\} = B$

$A \cap B = \{2, 3\}$

$A \cup B = \{1, 2, 3, 4, 5, \pi\}$

notation

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n$$

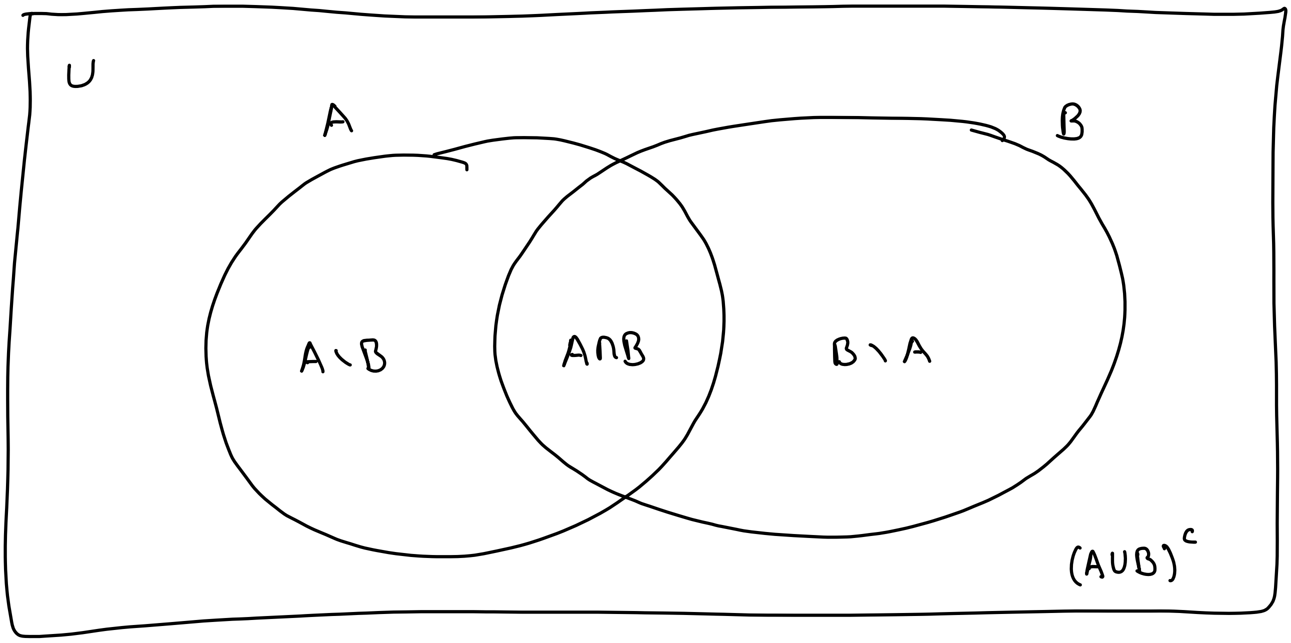
$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \dots \cap A_n$$

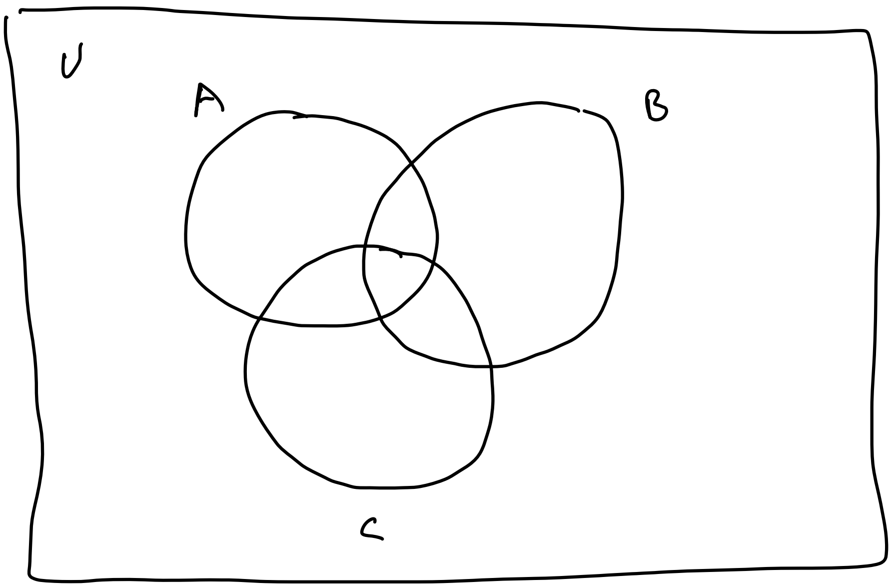
Def: The complement of a set A with respect to the superset U, denoted by A^c, is the set containing all element in U which are not in A.

Ex: Let $A = \{1, 2, 3\}$ and $U = \{2, 3, 5, 7, 1\}$

Here $A \subseteq U$. So $A^c = \{5, 7\}$.

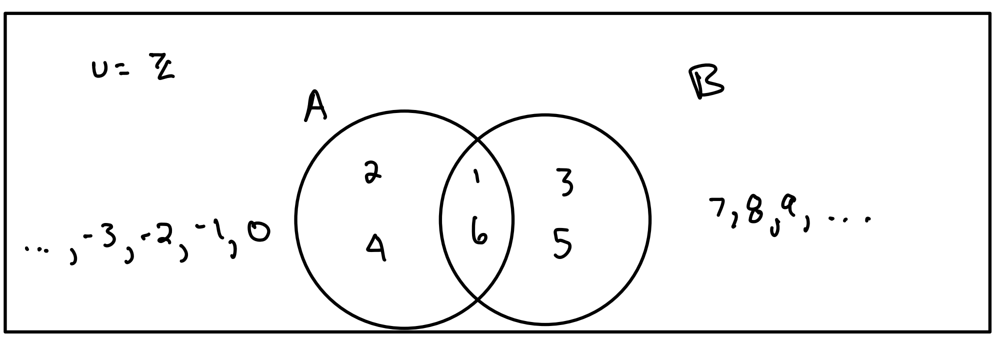
Venn diagram





Ex: Consider the following sets

$$A = \{1, 2, 4, 6\} \qquad B = \{1, 3, 5, 6\}$$



Notation:

$$A^n = \underbrace{A \times A \times A \times \cdots \times A}_{n\text{-times}}$$

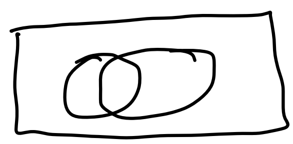
Ex:

$$A = \{1, 2\}$$

$$A^3 = \{(1,1,1), (1,1,2), (1,2,1), (1,2,2), (2,1,1), (2,1,2), (2,2,1), (2,2,2)\}$$

$$(1, (1,2)) \notin A^3$$

Thm: For any sets A and B,



$$(A \cup B)^c = A^c \cap B^c$$

pf: WTS $\underbrace{(A \cup B)^c \subseteq A^c \cap B^c}_{\text{case 1}}$ and $\underbrace{A^c \cap B^c \subseteq (A \cup B)^c}_{\text{case 2}}$

(case 1) Let $x \in (A \cup B)^c$. Hence $x \notin A \cup B$. Thus $x \notin A$ and $x \notin B$. Therefore $x \in A^c$ and $x \in B^c$ which implies $x \in A^c \cap B^c$. This shows

$$(A \cup B)^c \subseteq A^c \cap B^c$$

(case 2) Leave it to you!

$$A^c \cap B^c \subseteq (A \cup B)^c.$$