Reading Questions 4

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1. The expression "$x \in A$" means that x is an element of A. T
2. The set $\{x | x \in \mathbb{Z} \text{ and } x \geq 0\}$ contains all integers x such that x is greater than or equal to 0. T
3. The set N represents the set $\{0, 1, 2, \ldots\}$. T
4. Write out the elements of the set $\{a, b, \{a, b\}\}$. a, b, {a, b}

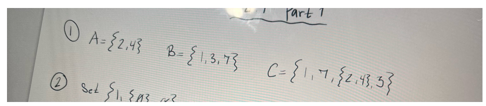
Section 2.1 Sets (Part 1)

Sets Notation

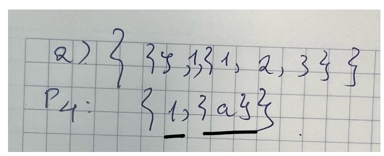
P 1. Write down a set named A which contains two elements. Write down a set named B which contains three elements. Write down a set named C which contains the set A and contains elements from the set B.

P 2. How many elements does the set $\{1, \{\emptyset\}, \emptyset\}$ contain.

P 3. Write out the elements of the following sets.



1. $\{x | x^2 + 2x - 3 = 0\}$ 1, -3
2. $\{\{\}, 1, \{1, 2, 3\}\}$ {}, 1, {1,2,3}



P 4. List the elements of the set $\{1, \{a\}, a\} \setminus \{a\}$?

Subsets

P 5. Let $A = \{1, 2, 3, 4\}$. List all the subsets B of A such that $B \subseteq \{1, 2\}$.

{}, {1}, {2}, {1, 2}

> **Definition**
>
> The *power set* of the set A, denoted by $\mathcal{P}(A)$, is the set of all subsets of A.

P 6. Write the power set $\mathcal{P}(A)$ for the set $A = \{\{1, 2\}, 3, \{\}\}$.

P 7. How many elements are in the power set of a set containing exactly three elements?

2^3

2.1 Set

Def: A set is a collection of elements.

Ex: $\{1,2,3\}$, $\{\{1,w\}, \pi, x^2+x\}$, $\{x \mid x^2=1\}$

are sets.

Def: The empty set is a set with no elements.

$$\emptyset = \{\ \}$$

Notation $\quad \underset{\text{in}}{\in}, \underset{\text{not}}{\notin}, \overset{\text{exclude}}{\setminus}, \underset{\text{equal}}{=}$

Ex: Let $A = \{1,2,3\}$ and $B = \{\{\}, 2, \{1,2,3\}\}$.

Then $1 \in A$, $A \in B$ and $B \setminus A = \{\emptyset, A\}$

Subsets

Def: A set A is a subset of the set B, denoted by $A \subseteq B$, if and only every element of A is in B.

In this case B is a superset of A. If A is not a subset of B then we write $A \nsubseteq B$.

Ex $\{1,2,3\} \subseteq \{1,2,3,4\}$ and

$\{\} \subseteq \{1,2\}$ and

$\{1,2\} \subseteq \{1,2\}$ and

$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$

Def: The power set of the set A, denoted by $\mathcal{P}(A)$, is the set of all subsets of A.

Ex: Let $A = \{1,2,3\}$. Then

$$\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

$$\mathcal{P}(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$$

Thm: For any set A, $A \subseteq A$ and $\{\} \subseteq A$.

Pf: (direct proof $A \subseteq A$)

Let $x \in A$. Since $x \in A$, $x \in A$. By def $A \subseteq A$.

(contradiction $\{\} \subseteq A$)

Assume $\{\} \not\subseteq A$. Then there exists $x \in \{\}$ such that $x \notin A$.

However this contradicts that $\{\}$ is empty. $\therefore$ $\{\} \subseteq A$.

__Def:__ $A = B$ iff $A \subseteq B$ and $B \subseteq A$.

__Ex:__ $\{1, 2\} = \{2, 1\}$