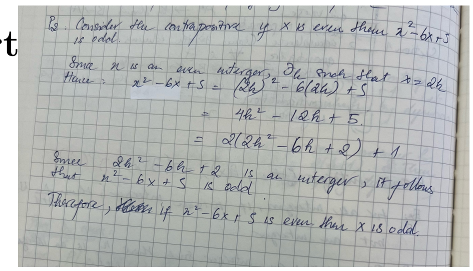
Reading Questions 3

page 13 Problem 5 and its solution

page 14 Problem 6 and its solution

1. To prove that the statement P is false by showing that the negation of P is true is a proof by contradiction. T
2. Consider the statement: "If the average of four different integers is 10 then one of the integers is greater than 11". This statement was proved using proof by contradiction. F
3. Were there any explanations since the previous class which were unclear?

Section 0.2 Proofs in Mathematics (Part



P2: Consider the contrapositive if x is even then x^2-6x+5 is odd.

Since x is an even integer, $\exists k$ such that $x=2k$

Hence: $x^2-6x+5 = (2k)^2 - 6(2k)+5$

$= 4k^2 - 12k + 5.$

$= 2(2k^2-6k+2)+1$

Since $2k^2-6k+2$ is an integer, it follows that x^2-6x+5 is odd.

Therefore, ~~Hence~~ if x^2-6x+5 is even then x is odd.

Proof by Contrapositive

P 1. When should proof by contrapositive be used to prove a statement?

P 2. Prove: If $x^2 - 6x + 5$ is even then x is odd.

Proof by Contradiction and Counter Example

P 3. When using proof by contradiction what is being contradicted?

P 4. Prove: No odd integer can be expressed as the sum of three even integers.

P 5. When is finding a counter example useful?

P 6. Prove or Disprove: For all positive integers x, if $\frac{x(x+1)}{2}$ is odd then $\frac{(x+1)(x+2)}{2}$ is odd.

contrapositive of $p \to Q$ is $\neg Q \to \neg P$

Thm: Let x be an integer. If $5x-7$ is even then x is odd.

pf: Consider the contrapositive (If x is even then $5x-7$ is odd.)

Since x is even, $\exists k$ - integer such that $x = 2k$. Hence

$$5x - 7 = 5(2k) - 7$$

$$= 10k - 8 + 1$$

$= 2(5k-4)+1.$

Since $5k-4$ is an integer it follows that $5x-7$ is odd.

Therefore the statement is true by way of contrapositive.

Contradiction $\neg P$ Assume $\neg P$ and find a contradiction

Thm: The real number $\sqrt{2}$ is irrational.

pf.:

P: $\sqrt{2}$ is irrational

$\neg P$: $\sqrt{2}$ is rational

WTS $\neg P$ is false.

Assume $\sqrt{2}$ is rational. Then $\exists\, m, n,\ n \neq 0$ s.t.

$\sqrt{2} = \frac{m}{n}$. Pick m and n in reduced form. In other words

m and n have no common factors.

Hence $2 = \frac{m^2}{n^2} \Rightarrow 2n^2 = m^2 \Rightarrow m^2$ - even

$\Rightarrow m$ - even

Thus $\exists\, k$ - integer s.t. $2 = \frac{(2k)^2}{n^2} \Rightarrow 2n^2 = 4k^2$

$\Rightarrow n^2 = 2k^2$

$\Rightarrow n^2$ - even

$\Rightarrow n$ - even

However m and n can not have the common factor of 2. Therefore $\sqrt{2}$ is not rational which implies $\sqrt{2}$ is irrational.

Counter Example

Ex: For all integers x, $x^2 > 0$.

when $x = 0$ $x^2 = 0 \not> 0$. Hence $x = 0$ is a counter example.