

# Section 0.2 Proofs in Mathematics (Part 1)

## Direct Proofs

**P 1.** Let  $x$  be an integer. Prove: If  $x$  is odd then  $x^2$  is odd. Write a sketch of the proof first. Then write a formal proof.

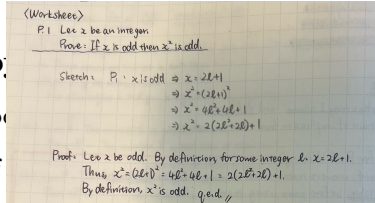
**P 2.** Let  $x$  be an integer. Prove: If  $x$  is even then  $x + 2$  is even.

**Proof by**

**P 3.** Prove: For all integers  $x$ ,  $x^2 - 3x + 9$  is odd.

**P 4.** Prove: The integer  $x$  is odd if and only if  $x^2$  is odd.

**P 5.** Prove: For all integer  $x$ ,  $x^2 \geq x$ .



0.2

Direct Proof } Today  
 Proof by Cases }

Proof by Contrapositive } Monday  
 Proof by Contradiction }  
 Proof by Counter example }

Direct Proof       $P_1 \rightarrow P_2 \rightarrow P_3 \rightarrow \dots \rightarrow P_n$

What I may assume

## Guidelines for Proof Writing

- write paragraphs
- define variable before using them
- transition words (thus, hence, then, therefore, etc)
- start each statement with an English word

Thm: Let  $x$  be an integer. If  $x$  is even then  $x^2$  is even.

sketch:      want to show  
                     $\downarrow$   
WTS:       $\exists k$  an integer s.t.  $x^2 = 2k$

$$\begin{aligned} P_1: x \text{ is even} & \Rightarrow 2l = x \\ & \Rightarrow (2l)^2 = x^2 \\ & \Rightarrow 4l^2 = x^2 \\ & \Rightarrow 2(2l^2) = x^2 \end{aligned}$$

pf: Let  $x$  be even. By definition, for some integer  $l$ ,  $x = 2l$ .  
Hence  $x^2 = (2l)^2 = 2(2l^2)$ . Let  $t = 2l^2$ . Then  $x^2 = 2t$ .  
since  $t$  is an integer by definition  $x^2$  is even.      qed



## Proof by Cases

Case  $P(x_1), P(x_2), \dots, P(x_n)$

Then: For all integers  $x$ ,  $x^2+x$  is even.

Let  $x$  be a real number. If  $x$  is an integer then  $x^2+x$  is even.

(case 1:  $x$  is even)

Assume  $x$  is even, Then  $\exists l$  - an integer such that  $x=2l$ . Hence

$$x^2+x = (2l)^2 + 2l = 2(2l^2 + l). \text{ Therefore } x^2+x \text{ is even.}$$

(case 2:  $x$  is odd)

Assume  $x$  is odd. Then  $\exists l$  - an integer such that  $x=2l+1$ .

Hence

$$\begin{aligned} x^2+x &= (2l+1)^2 + (2l+1) \\ &= 4l^2 + 4l + 1 + 2l + 1 \\ &= 2(2l^2 + 3l + 1). \end{aligned}$$

Therefore  $x^2+x$  is even.