MW 3p - 4p   R 1:30p - 2:30pm

Reading Questions 2

page 10 Pause 10

$P \to R$ and $R \to P$

page 12 Problem 1

$Q \to R$ $R \to Q$

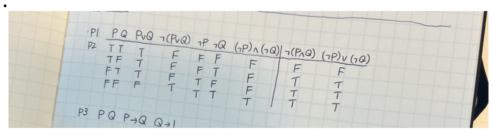
1. The statements P and Q are equivalent if $P \iff Q$. ~~F~~ T
2. If $P \to Q \to R \to P$ is a true statement then P, Q, and R are equivalent statements. ~~F~~ T $\underbrace{R \to P \to Q}$ $\Downarrow$ $R \to Q$
3. Rewrite the statement "A matrix with determinant 1 is invertible." as an implication.

$R \to P \to Q$ If a matrix has det = 1 then it is invertible.

Section 0.1 Compound Statements (Part 2)

More on Truth Values

P 1. In some cases, proving an equivalent statement may be easier than proving the actual statement. Determine if the following statements are equivalent.



P Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$(\neg P) \wedge (\neg Q)$	$\neg(P \wedge Q)$	$(\neg P) \vee (\neg Q)$
T T	T	F	F	F	F	F	F
T F	T	F	F	T	F	T	T
F T	T	F	T	F	F	T	T
F F	F	T	T	T	T	T	T

P3 P Q $P \to Q$ $Q \to$

$$\neg(P \vee Q) \iff ((\neg P) \wedge (\neg Q))$$

P 2. Determine if the following statements are equivalent.

$$\neg(P \wedge Q) \iff ((\neg P) \vee (\neg Q))$$

P 3. Use a truth table to show that $\big[(P \to Q) \wedge (Q \to R)\big] \to (P \to R)$.

More Compound Statements

P 4. Math symbols are useful when sketching a proof. Use the symbols $\neg, \to, \forall$ (for all) and $\exists$ (there exists) to transcribe the following statements into logical notation.

1. If $y = 1$, then $xy = x$ for any x.
2. There is no solution to $x^2 = y$ unless $y > 0$.
3. $x < z$ is a necessary condition for $x < y$ and $y < z$.
4. If $x < y$ then for some z such that $z < 0$, $xz > yz$.
5. There is an x such that for every y and z, $xy = xz$.

P 5. Negations are often used to show that a statement is false. Write the negation of the following statement.

R : If $\underbrace{\text{The integer 3 is even}}_{P}$, then $\underbrace{9^2 \text{ is even}}_{Q}$.

P 6. Write the contrapositive of the implication $Q \Rightarrow P$ from the previous problem.

Def: P and Q are equivalent if they have the same truth table.

Ex:

P	Q	$\neg P$	$\neg Q$	$P \to Q$	$\neg Q \to \neg P$
T	T	F	F	T	T
F	T	T	F	T	T
T	F	F	T	F	F
F	F	T	T	T	T

Ex: Let x be an integer.

$$\text{If } \underbrace{x \text{ is even}}_{P} \text{ then } \underbrace{x^2 \text{ is even}}_{Q}.$$

$$\text{If } \underbrace{x^2 \text{ is old}}_{\neg Q} \text{ then } \underbrace{x \text{ is old}}_{\neg P}.$$

Def: Quantifiers are expressions that quantify statements.

Notation for all $\forall$

there exists $\exists$

Ex: 1) For all integers x, $x^2 \geq 0$.

Let x be an integer.

2) There exists an integer x such that $x^2 - 1 = 0$.

$\exists_x$. s.t. $x^2 - 1 = 0$

Let x and y be real numbers.

3) $\forall_x \; \exists_y$ such that $x < y$.

Ex: Let P: The number 8 is greater than 0.

Then $\neg P$: The number 8 is less than or equal to 0.

Ex: Let P: The value of x is greater than 0.

Then $\neg P$ is not: The value of x is less than or equal to 0.

Ex: Let x be a real number.

Let P: The value of x is greater than 0.

$\neg P$: The value of x is less than or equal to 0.

Def: $\neg Q \to \neg P$ is the contrapositive of $P \to Q$.

Ex: Let x be a real number. The contrapositive of

"If $\overbrace{x > 0}^{P}$ then $\overbrace{x^2 > 0}^{Q}$" is

"If $\underbrace{x^2 \le 0}_{\neg Q}$ then $\underbrace{x \le 0}_{\neg P}$".