Section 0.1 Compound Statements (Part 1)

P 1. The truth value of a statement in an implication may be determined by the truth value of the implication. Given that p and $p \to q$ are true, determine the truth value of q.

P 2. Given that $\neg p$ and $p \vee q$ are true, determine the truth value of q.

Section 0.1 Compound Statements (Part 2)

P 3. In some cases, proving an equivalent statement may be easier than proving the actual statement. Determine if the following statements are equivalent.

$$\neg(P \vee Q) \iff ((\neg P) \wedge (\neg Q))$$

P 4. Determine if the following statements are equivalent.

$$\neg(P \wedge Q) \iff ((\neg P) \vee (\neg Q))$$

P 5. Use a truth table to show that $\big[(P \to Q) \wedge (Q \to R)\big] \to (P \to R)$.

Section 0.2 Proofs in Mathematics (Part 1)

P 6. Let x be an integer. Prove: If x is even then $x + 2$ is even.

P 7. Prove: For all integers x, $x^2 - 3x + 9$ is odd.

P 8. Prove: The integer x is odd if and only if x^2 is odd.

Section 0.2 Proofs in Mathematics (Part 2)

P 9. Prove: If $x^2 - 6x + 5$ is even then x is odd.

P 10. Prove: No odd integer can be expressed as the sum of three even integers.

P 11. Prove or Disprove: For all positive integers x, if $\frac{x(x+1)}{2}$ is odd then $\frac{(x+1)(x+2)}{2}$ is odd.

Section 2.1 Sets (Part 1)

P 12. Write out the elements of the following sets.

1. $\{x | x^2 + 2x - 3 = 0\}$
2. $\{\{\}, 1, \{1, 2, 3\}\}$

P 13. List the elements of the set $\{1, \{a\}, a\} \setminus \{a\}$?

P 14. Let $A = \{1, 2, 3, 4\}$. List all the subsets B of A such that $B \subseteq \{1, 2\}$.

P 15. Write the power set $\mathcal{P}(A)$ for the set $A = \{\{1, 2\}, 3, \{\}\}$.

Section 2.2 Operations on Sets (Part 1)

P 16. Let $A = \{a, b, c\}$ and $B = \{A, b, 3\}$. Find $A \cup B$ and $A \cap B$.

P 17. What is the complement of the set $\{1, 3, 9, 27\}$ with respect to the set

$$\{1, 2, 3, 4, 8, 9, 16, 27, 32, 81\}?$$

P 18. Make a Venn diagram for the sets $A = \{1, 2, 3\}$, $B = \{1, 4, 5\}$, and $C = \{2, 5, 7\}$.

P 19. Let $A = \{1, 2\}$ and $B = \{x, y, z\}$. Find $B \times A$ and B^2.

P 20. Prove that for any sets A and B, $(A \cap B)^c = A^c \cup B^c$.

Section 3.1 Basic Terminology (Part 1)

P 21. Is the set $\{(1, 2), (3, 1), (2, 1)\}$ a function from the set $\{1, 2, 3\}$ to the set $\{1, 2, 3\}$.

P 22. Let $A = \{1, 2, 3\}$ and $B = \{a, b, c, d\}$. Give an example of a relation from A to B containing exactly three elements such that the relation is not a function from A to B.

P 23. Let $A = \{a, b, c, d\}$ and $B = \{x, y, z\}$. Then $f\{(a, y), (b, z), (c, y), (d, z)\}$ is a function from A to B. Determine $\operatorname{dom} f$ and $\operatorname{rng} f$.

P 24. Let $A = \{w, x, y, z\}$ and $B = \{r, s, t\}$. Give an example of a function $f : A \to B$ that is neither one-to-one nor onto.

P 25. Show that the function $f = \{(x_1, x_2) \,|x_1^2 = x_2\}$ from $\mathbb{N}$ to $\mathbb{N}$ is one-to-one. Is the function onto?

P 26. Let $f = \{(x, y)|y = 3x + 5\}$ be the function from $\mathbb{N}$ to $\mathbb{N}$. Show that f a bijective.

Section 3.2 Inverse and Composition (Part 1)

P 27. Find the inverse relation of the following relation.

$$R = \{(1, 8), (3, 3), (4, 3), (2, 1), (5, 2)\}$$

P 28. Determine if the following functions from the set $\{1, 2, 3, 4, 5\}$ itself have an inverse. If so find the inverse.

$$f = \{(1, 3), (3, 4), (4, 3), (2, 1), (5, 2)\} \quad \text{and} \quad g = \{(1, 2), (3, 1), (2, 4), (4, 3), (5, 5)\}$$

P 29. Let $A = \{1, 2, 3, 4\}, B = \{a, b, c, d\}$ and $C = \{r, s, t, u, v\}$ and define the functions $f : A \to B$ and $g : B \to C$ by

$$f = \{(1, b), (2, d), (3, a), (4, a)\} \text{ and } g = \{(a, u), (b, r), (c, r), (d, s)\}.$$

Determine $g \circ f$ and $(g \circ f)(1)$.

P 30. Let $f : A \to A$ and $g : A \to A$ be functions. Show that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

P 31. Let $f : A \to B$ and $g : B \to C$ be functions. Prove that if $g \circ f$ is one-to-one and f is onto then g is one-to-one.

Section 3.3 One-to-One Correspondence and the Cardinality of Sets (Part 1)

P 32. Is there a one-to-one correspondence from $\{1, 2, 3, \ldots, n\}$ to the empty set? Is the set finite or infinite?

P 33. Show that the set $\{x, y, z, 1\}$ is a finite set.

P 34. Show that the set $\mathbb{N} \cup \{0\}$ is a countable set.

P 35. Determine if the set $\{3^n | n \in \mathbb{Z}\}$ is countable.