Section 9.4 Tests for Convergence (Part 2)

Limit Comparison

Theorem: Limit Comparison Test

Suppose $a_n > 0$ and $b_n > 0$ for all n. If

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c$$

where $c > 0$ then the two series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge or both diverge.

P 1. Use the Limit Comparison Test to determine if the series $\sum_{n=1}^{\infty} \frac{n+5}{n^2+4}$ converges or diverges. In general, you may use any test to determine if a series converges or diverges.

P 2. Use the Limit Comparison Test to determine if the series $\sum_{n=1}^{\infty} \frac{n+2}{n^4+n+1}$ converges or diverges.

Absolute Value Test

Theorem: Absolute Value Test

If $\sum_{n=1}^{\infty} |a_n|$ converges, then so does $\sum_{n=1}^{\infty} a_n$.

P 3. Use the absolute value test to determine if the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2+1}$ converges or diverges.

P 4. Determine if the series $1 - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \cdots$ converges or diverges.

P 5. Is the statement "If $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} |a_n|$ converges" always true. Explain your answer.

P 6. Suppose $|a_n| < ax^n$ where $|a_i| < a$ and $0 < x < 1$. Does the series $\sum_{n=1}^{\infty} a_n$ converge or diverge?