Reading Questions 12

1. The improper integral $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$ is the limit of some sequence.

2. The sequence $\int_1^b \frac{1}{\sqrt{x}}\,dx$, grows without bound as $b \to \infty$.

3. Does $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$ converge or diverge?
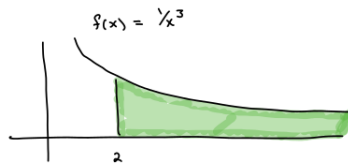
Section 7.6 Improper Integrals (Part 1)

Type I

Theorem: Improper Integral I

Suppose $f(x)$ is positive for $x \geq a$. If $\lim_{b \to \infty} \int_a^b f(x)\,dx$ converges then

$$\int_a^\infty f(x)\,dx = \lim_{b \to \infty} \int_a^b f(x)\,dx$$

and the integral converges. Otherwise $\int_a^\infty f(x)\,dx$ diverges.

P 1. Write down an integral that represents the area of the shaded region in the figure below.

P 2. Determine if $\int_0^\infty \frac{e^x}{1+e^x}\,dx$ converges or diverges.

P 3. Determine if $\int_2^\infty \frac{3}{x^3}\,dx$ converges or diverges.

Type II

P 4. Determine if $\int_{-\infty}^\infty \frac{1}{x^2+25}\,dx$ converges or diverges.

Theorem: Improper Integral II

Suppose $f(x) > 0$ and continuous on $a \leq x < b$ and tends to infinity as $x \to b$. Then

$$\int_a^b f(x)\,dx = \lim_{c \to b^-} \int_a^c f(x)\,dx$$

and the integral converges. Otherwise the integral diverges.

P 5. Determine if $\int_3^6 \frac{1}{(4-x)^2}\,dx$ converges or diverges.