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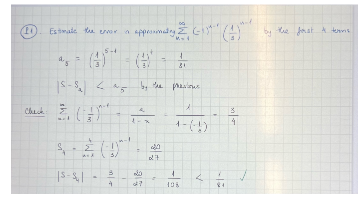
Reading Questions 6

reading: Section 9.4 : Subsection - Absolute and Conditional Convergence

1. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ converges conditionally. T

2. A series $\sum_{n=1}^{\infty} a_n$ converges absolutely if $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} |a_n|$ both converge. T



Section 9.4 Tests for Convergence (Part 4)

Approximation

> **Theorem**
>
> Let S_n be the nth partial sum of the series $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ and let $S = \lim_{n\to\infty} S_n$. Suppose $0 < a_{n+1} < a_n$ for all n and $\lim_{n\to\infty} a_n = 0$. Then $|S - S_n| < a_{n+1}$.

P 1. Estimate the error in approximating the sum of the series $\sum_{n=1}^{\infty} (-1)^{n-1} (\frac{1}{3})^{n-1}$ by the sum of the first 4 terms. Check your answer.

// $S_4 =$

P 2. Let $S = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$. Give a number k such that $|S - \frac{31}{36}| < k$.

Absolute Convergence

> **Definition**
>
> The series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent if both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} |a_n|$ converge.

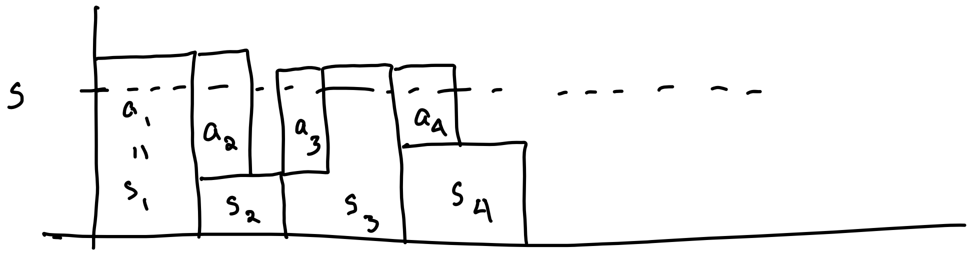
> **Definition**
>
> The series $\sum_{n=1}^{\infty} a_n$ is conditionally convergent if $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ diverges.

P 3. Determine if the series $\sum_{n=1}^{\infty} (-1)^{n-1} (\frac{2}{3})^n$ is absolutely or conditionally convergent.

P 4. Determine if the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{\sqrt{n}}$ is absolutely or conditionally convergent.

9.4 part 4 Approximations

Suppose $\lim_{n\to\infty} S_n = S = \sum_{i=0}^{\infty} a_i$ $\qquad S_n = \sum_{i=0}^{n-1} a_i$



<u>Thm:</u> Let S_n be the n^{th} partial sum of $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ and

let $S = \lim_{n\to\infty} S_n$. Suppose $0 < a_{n+1} < a_n$ for all n

and $\lim_{n\to\infty} a_n = 0$. Then $|S - S_n| < a_{n+1}$.

<u>Ex:</u> Estimate the error in approximating $\sum_{n=1}^{\infty} (-1)^{n-1} \underbrace{\left(\frac{1}{2}\right)^{n-1}}_{a_n}$

by the sum of the first 4 terms. Here $a_5 = \left(\frac{1}{2}\right)^{5-1}$

$= \frac{1}{16}$

$\underbrace{|\overset{\text{exact}}{S} - \overset{\text{appr.}}{S_4}|}_{\text{error}} < a_5$ by the previous

<u>Check:</u> $\sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^{n-1} = \frac{1}{1-\left(-\frac{1}{2}\right)} = \frac{2}{3}$

$S_4 = \sum_{n=1}^{4} \left(-\frac{1}{2}\right)^{n-1} = \frac{5}{8}$ check

$|S - S_4| = \left|\frac{2}{3} - \frac{5}{8}\right| = \frac{1}{24} < \overset{a_5}{\overset{\parallel}{\frac{1}{16}}}$ as $16 < 24$

<u>Def:</u> if $\sum a_n$ and $\sum |a_n|$ converge

then $\sum a_n$ converges absolutely

<u>Def:</u> if $\sum a_n$ converges and $\sum |a_n|$ diverges

then $\sum a_n$ converges conditionally.

<u>Ex:</u> $\sum (-1)^{n-1} \frac{1}{n}$ converges conditionally as

$\underbrace{\sum \frac{1}{n}}_{\text{p-series}}$ diverges and $\sum (-1)^{n-1} \frac{1}{n}$ converges

by alternating series test

<u>Ex:</u> $\sum (-1)^{n-1} \frac{1}{n^2}$ converges absolutely as both

$\underbrace{\sum \frac{1}{n^2}}_{\text{p-series}}$ and $\underbrace{\sum (-1)^{n-1} \frac{1}{n^2}}_{\text{alt series test}}$ converge