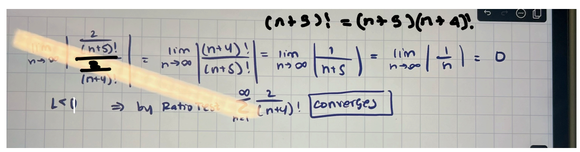
Reading Questions 5

1. The value of 4! is 20. F
2. If n is a positive integer then $(n+1)! = n!(n+1)$. T
3. The series $\sum_{n=1}^{\infty} \frac{1}{n!}$ diverges. F
4. Which test was used in the example? Ratio Test

Section 9.4 Tests for Convergence (Part 3)

Ratio Test

> **Theorem: Ratio Test**
>
> Suppose $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = L$. then the series $\sum_{n=1}^{\infty} a_n$
>
> 1. converges if $L < 1$,
> 2. is undetermined if $L = 1$, and
> 3. diverges otherwise.



P 1. Up until this section, the sequence a_n of a series has not contained a factorial. Hence if the sequence contains a factorial that might be an indication to use the Ratio Test.

Determine if the series $\sum_{n=1}^{\infty} \frac{2}{(n+4)!}$ converges or diverges.

P 2. Can the Ratio Test be used to determine if the series $\sum_{n=1}^{\infty} (-1)^{n-1}\frac{1}{n}$ converges or diverges?

P 3. Determine if the series $\sum_{n=1}^{\infty} \frac{6}{n+2^n}$ converges or diverges. Don't forget to first determine if $\lim_{n\to\infty} a_n = 0$.

Alternating Series

> **Theorem: Alternating Series Test**
>
> The series $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ converges if $0 < a_{n+1} < a_n$ and $\lim_{n\to\infty} a_n = 0$.

P 4. Determine if the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+1}$ converges or diverges. Be sure to state any test that you use.

P 5. Can the alternating series test be used to determine if the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n!}{n^3+2}$ converges or diverges? If so, state whether the series converges or diverges.

P 6. The alternating series test can be applied to $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ where $0 < a_{n+1} < a_n$ for all n and $\lim_{n\to\infty} a_n = 0$. Give an example of a sequence a_n where $0 < a_{n+1} < a_n$ for all n and $\lim_{n\to\infty} a_n \neq 0$.

9.4

geometric series	$\sum a x^n$
monotone and bounded	$\nearrow$ $\searrow$
divergent test	$\lim a_n \neq 0$
comparison test	$a_n < b_n$
p-series test	$\sum \frac{1}{n^p}$
limit comparison test	$\lim \frac{a_n}{b_n} = L > 0$
absolute value test	$\sum \lvert a_n \rvert$

Ratio Test

Thm: Suppose $\lim\limits_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$ then $\sum\limits_{n=1}^{\infty} a_n$

1. converges if $L < 1$
2. is undetermined if $L = 1$, and
3. diverges otherwise

Ex $a_n = 1, 1, 1, 1 \ldots$ $\sum 1$ diverges

$\lim \left| \frac{1}{1} \right| = 1$ The ratio test doesn't give any information.

Ex: Consider $\sum\limits_{n=1}^{\infty} \overset{a_n}{\overset{\shortparallel}{\frac{1}{n!}}}$.

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{\frac{1}{(n+1)!}}{\frac{1}{n!}} \right| = \lim_{n\to\infty} \left| \frac{n!}{(n+1)!} \right| = \lim_{n\to\infty} \left| \frac{n!}{(n+1)\, n!} \right|$$

$$= \lim_{n\to\infty} \left| \frac{1}{n+1} \right| = 0 < 1$$

By the ratio test $\sum \frac{1}{n!}$ converges.

Ex: consider $\sum \frac{1}{n}$.

$$\lim_{n\to\infty} \left| \frac{\frac{1}{n+1}}{\frac{1}{n}} \right| = \lim_{n\to\infty} \left| \frac{n}{n+1} \right| = 1.$$

The ratio test is inconclusive.

$$a_n = \frac{6}{n+2^n} \qquad a_{n+1} = \frac{6}{(n+1)+2^{n+1}} \qquad \frac{a_{n+1}}{a_n} = \frac{\frac{6}{(n+1)+2^{n+1}}}{\frac{6}{n+2^n}}$$

$$= \frac{n+2^n}{(n+1)+2^{n+1}}$$

$$\lim \cancelto{0}{\frac{n}{(n+1)+2^{n+1}}} + \lim \cancelto{0}{\frac{2^n}{(n+1)+2^{n+1}}} = 0 < 1$$

Alternating Series

$$\underbrace{\sum (-1)^n b_n}_{a_n} \qquad b_n > 0$$

Thm: $\sum (-1)^{n-1} b_n$ converges if $0 < b_{n+1} < b_n$ and

$$\lim b_n = 0.$$

Ex: Consider $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$.

Note: the absolute value test fails.

$a_n = (-1)^{n-1} \frac{1}{n}$ $\quad b_n = \frac{1}{n}$ $\quad b_{n+1} = \frac{1}{n+1}$

$\frac{1}{n+1} < \frac{1}{n}$ as $n < n+1$

$\lim b_n = \lim \frac{1}{n} = 0$

By the alternating series test $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$ converges.